Altimeter Crossover Methods for Precision Orbit Determination and the Mapping of Geophysical Parameters

C. K. Shum, B. H. Zhang, B. E. Schutz and B. D. Tapley

Abstract

Methodologies to use differenced height measurements at the points where the ground tracks of the altimetric satellite orbits intersect (crossover measurements) have been developed for the purpose of precise orbit determination and recovery of geophysical parameters of interest. Analysis has indicated that the inclusion of single-satellite crossover data contribute significantly to the improvement of the Earth's gravity field model. A new technique using crossover measurements from two satellites carrying altimeter instruments for precise orbit determination has been developed. For altimetric satellites with distinct orbits, the dual-satellite altimeter crossover technique is found to be capable of observing zonal harmonics of the Earth's geopotential which are weakly observed using single-satellite crossovers. Also developed are techniques to efficiently calculate the single- and dual-satellite crossover locations and to generate the crossover measurements from the altimeter data. Depending on the orbit characteristics, dual-satellite crossover measurements are significantly more sensitive to the geopotential perturbations than single-satellite crossover measurements. In particular, order one (m-daily) geopotential coefficients using dual-satellite crossovers are found to be more sensitive than those of single-satellite crossovers. The dual-satellite crossover technique will provide an important means for precise orbit determination and recovery of geophysical parameters using crossover data between the future oceanographic satellites, such as TOPEX/POSEIDON and ERS-1.

Introduction

The ability of satellite-borne radar altimeter data to measure the global ocean surface with subcentimeter precision provides a unique data set for precision orbit determination and the mapping of the Earth's gravity field and its marine
geoid. Although satellite altimetry has a distinct advantage in its global distribution of data, consideration of possible error sources when altimeter data are used for orbit determination and geophysical mapping reveals several disadvantages. In particular, oceanographic features and nontemporal ocean topography can be absorbed into orbit and geodetic parameters when altimeter data are used directly. The nontemporal ocean topography, mostly due to error in the marine geoid, currently has an uncertainty of several meters in some areas and is significant when decimeter radial orbit accuracy of certain satellite orbits is desired. A technique which eliminates the altimeter's dependence on the nontemporal ocean topography is the use of altimeter measurements at the points where the orbit ground tracks intersect. These points are referred to as crossover points. A crossover measurement is defined as the difference in the altimeter measurements at the points where the satellite ground track intersects. Since the constant part of each altimeter measurement will be partly comprised of the marine geoid and the quasi-stationary sea surface topography, the crossover or differenced altimeter measurements will be a function of the orbit variations and the time-varying ocean surface topography. Mismodeling of the forces which act on the satellite will be the primary contribution to the residuals of the crossover measurements if the error in the modeling of the time-varying topography is small. The orbit error is primarily a long wavelength feature with once-per-orbital-revolution (40,000 km wavelength) and twice-per-orbital-revolution (20,000 km wavelength) components dominating. Although the nontemporal portion of the ocean topography can be eliminated at the crossover point, the remaining temporal changes as well as altimeter time tag error, can still be aliased into the radial orbit error on a global basis.

Global analysis of crossover residuals can provide valuable information about radial orbit error sources and the recovery of interesting geophysical parameters. The use of oceanographic and geophysical applications of satellite altimetry was described by Shum [1] and Douglas et al. [2]. The use of altimeter crossover points to evaluate the accuracy of SEASAT ephemerides has been proven valuable, for example, by Shum [1] and by Schutz et al. [3]. Crossover residuals have also been used for validation of the SEASAT altimeter time tag accuracy (Marsh and Williamson [4]; Schutz et al. [5]). Crossover techniques are also used in gravity anomaly recovery (Rummel and Rapp [6]), in the recovery of ocean tide constituents (Parke [7]; Brown [8]), and in the analysis of oceanic eddy variability (Cheney et al. [9]).

Geometrical, or non-dynamical, crossover techniques have been used to remove long wavelength orbit error, for example, by Sandwell et al. [10]. These techniques have also been applied to the generation of maps of global mean sea surfaces (Marsh et al. [11]) and global mesoscale variability (Cheney et al. [9]). Hagar and Boggs [12] and Shum [1] investigated the observability characteristics of single-satellite altimeter crossover measurements in their application to dynamic orbit determination. Dynamical techniques to use altimeter and altimeter crossover data have been applied to obtain an improved gravity field model for SEASAT (Shum [1]; Schutz et al. [3]). Santee [13] and Born et al. [14] performed a study to use geometrical techniques to improve the orbit of the proposed Navy satellite, N-ROSS, with the use of altimeter crossover data between N-ROSS and
the proposed U.S.-French oceanographic satellite mission, TOPEX/POSEIDON. Shum et al. [15] examined the observability of geopotential coefficients using dual-satellite crossover data in an orbit determination procedure. More recently, the University of Texas preliminary TOPEX gravity fields have included altimeter and altimeter crossover data from SEASAT and GEOSAT (Tapley et al. [16]).

The TOPEX/POSEIDON mission, which is conducted jointly by the National Aeronautics and Space Administration (NASA) and France's Centre Nationale d'Etudes Spatiale (CNES), will carry precise radar altimeter instruments for the measurement of oceanic topography including the mean and variable surface geostrophic currents and tides of the world's oceans, and to lay the foundation for a continuing program to provide long-term observations of the general ocean circulation and its variability. In the same time span during the early 1990's, the European Space Agency (ESA) will launch another altimeter satellite, ERS-1. Since ERS-1 will cover most of the ocean surface with its 98.5° inclination and the TOPEX/POSEIDON mission will provide a more accurate radial orbit (13 cm) (Tapley et al. [17]), the use of crossover measurements between the satellites will provide a significant means for precise determination and the mapping of geophysical and oceanographic phenomena. This paper describes the techniques to apply single- and dual-satellite altimeter crossover data for the purpose of precision orbit determination and the estimation of geophysical parameters.

Crossover Measurement Geometry

The intersections of the ground track of the subsatellite point for an Earth-orbiting satellite on the surface of the Earth, referred to as the “crossover” points, are caused by rotation of the Earth and, to a lesser extent, non-Keplerian perturbations of the satellite's orbit. Assuming the crossing occurs at times $t_i$ and $t_j$, where $t_i < t_j$, and that the radar altimeter measurements, $h(t_i)$ and $h(t_j)$, are referred to the reference ellipsoid and made precisely at these two times, a crossover measurement, $\Delta h(t_i, t_j)$, can be computed as follows (Fig. 1):

$$\Delta h(t_i, t_j) = h(t_i) - h(t_j)$$  

The altimeter measurement model defined upon the reference ellipsoid can be described as follows:

$$h(t) = h_s - h_{\ell} + (h_a - h_c + h_r + b + \eta)$$

where $h(t)$ is the instantaneous altimeter height measurement at time $t$ between the altimeter instrument and the point of radar pulse contact on the instantaneous ocean surface; $h_s$ is the computed altimeter measurement between the spacecraft center of mass and the reference ellipsoid; $h_{\ell}$ is the sea surface height above the reference ellipsoid at a specified geographical location, which includes the contribution due to the geopotential, the constant and dynamic sea surface topography, the ocean and solid Earth tides caused by the Sun and Moon, and other atmospheric and oceanographic phenomena; $h_a$ is the correction for atmospheric propagation delay; $h_c$ is the correction for the location of the spacecraft center of mass and the electronic reference point to which $h$ is referred; $h_r$, is the
relativistic effect associated with the gravitational field of the Earth, \( b \) is the correction for a possible bias in \( h \); and \( \eta \) is the unmodeled errors in the mathematical model, both systematic and random. In view of equation (1), the crossover measurement model thus becomes

\[
\Delta h(t_i, t_j) = \Delta h_i - \Delta h_g - \Delta h_e - \Delta h_s + \Delta h_r + \Delta b + \Delta \eta
\]

where \( \Delta \) denotes the difference in the values defined at times \( t_i \) and \( t_j \). The time tags \( t_i \) and \( t_j \) can be those of a single satellite or of two different satellites.

Note that any nontemporal or very long wavelength ocean phenomena, that are relatively invariant during the time interval \( t_i \) to \( t_j \), will be eliminated. In particular, the constant or very long wavelength part of \( \Delta h_g \) due to the geoid and \( \Delta b \) will be identically zero in equation (3). Slow time-varying quantities compared to the time interval associated with the two crossover times will also vanish. Since the ocean surface is dynamic, \( \Delta h(t_i, t_j) \) includes a contribution which represents the temporal change in the ocean surface height at the crossover point.

For the case of altimeter crossovers from two different satellites carrying radar altimeters, \( t_i \) and \( t_j \) in equation (3) denote time tags for satellites \( i \) and \( j \), respectively. The geographically correlated content of \( h_n \), due to the orbit error, which would be eliminated for single-satellite crossovers, will be different at the crossover point for satellite orbits with distinct altitudes and inclinations. The zonal harmonics which comprise part of the geographically correlated gravity error are insensitive to crossover data collected from a single satellite in a near-circular orbit (Rosborough [18]). However, satellite crossover data, generated from two satellites with different inclinations and altitudes, will be sensitive to the zonal harmonics.

It is rare, that altimeter measurements are available precisely at the crossover times. In practice, a “pseudo measurement” is formed by interpolating from available altimeter measurements in the vicinity of the crossover times.
Since the orbit will not be perfect, the actual computed crossover measurement is

$$\Delta h^*(t_i, t_j) = h^*(t_i) - h^*(t_j)$$

where ($\cdot$)* denotes that the altimeter measurements are computed using nominal reference orbits. Hence, $\Delta h^*(t_i, t_j)$ in equation (4) contains a contribution to the radial orbit error of unmodeled forces acting on the satellite, as well as unmodeled temporal changes in the ocean topography.

**Crossover Measurement Computation**

As described in the preceding section, the location of crossovers are functions of two time tags, $t_i$ and $t_j$, which depends on the satellite ephemerides. Analytic theory has been developed to predict locations and time tags of single- and dual-satellite crossovers (Shum [1]; Santee [13]). One of the advantages of the analytic theory is that it provides approximate crossover locations which can be improved using an ephemeris determined by tracking data. However, the theory to predict crossover solutions, especially in the case of a general dual-satellite scenario, is complicated, and depending on the characteristics of the orbit geometry, may not yield accurate crossover locations.

A generalized single- and dual-satellite crossover measurement computation software system has been developed to compute single- and dual-satellite crossover time tags, locations and interpolated altimeter measurements at crossover locations. Geophysical Data Records (GDR) which consist of altimeter measurements, sea surface heights and nominal orbits predicting the location of the satellite are used as input to the software system. Optional input can include user-supplied orbits or mean orbital elements for the satellites. Data are grouped into direct access disk storage files by satellites and by their respective orbital revolutions. Approximate crossover locations and their time tags are first determined from the intersections of two orbital revolutions which are modeled by cubic spline fits. Cubic splines were used because of the smooth characteristics of orbital heights which allow the use of one set of computationally efficient cubic spline coefficients for each orbital revolution (Zhang [19]). Altimeter inferred sea surface height measurements are then interpolated to obtain more accurate crossover locations and time tags using a 10-second window centered at the approximate crossover locations. The sea surface height scheme employed is a least square cubic spline which is iterated as bad data points are edited. Figure 2a shows a relatively good case where the cubic spline fit of the observed sea surface heights has a root-mean-squared (rms) residual of 1.5 cm. Figure 2b shows a poor case with a residual of 9.4 cm. For the computation of the SEASAT and GEOSAT crossover measurements, altimeter data with a cubic spline fit residual greater than 9 cm were edited. The two height measurements necessary for a crossover are then computed by interpolating the altimeter measurements at the crossover times.

Figure 3a shows the locations of single-satellite crossovers for a 10-day ERS-1 orbit which has an inclination of 98.5° and an altitude of 800 km. Figure 3b
shows the locations of dual-satellite crossovers for a 3-day ERS-1 orbit and a 10-
day TOPEX orbit which has an inclination of 63.4° and an altitude of 1300 km. Excellent geographical coverages are seen in both cases.

Gravity Field Solution Methodology Using Altimeter Crossovers

Procedures required to use single- and dual-satellite crossover measurements for orbit determination and geophysical parameters reduction are different from

FIG. 3b. Dual-Satellite Crossovers for TOPEX and ERS-1: 10-Day Orbits, 22,783 Crossovers Over Ocean.

the processing of conventional ground-based tracking data. Several procedures which are unique to crossover measurements must be employed. The description of these procedures given below can be adapted to most orbit determination computer programs, however, the description pertains to the University of Texas Orbit Processor (UTOPIA).
Step 1: Crossover measurements for satellites $i$ and $j$ are computed from nominal orbits using the procedures described in the preceding section. The nominal orbits may have been previously determined from a least-squares estimate using ground-based tracking data. Crossover measurements are stored in time sequential order, and the crossover occurrences are referenced by predetermined address reference indices.

Step 2: Ephemerides are propagated for satellites $i$ and $j$ and other non-crossover measurements are processed sequentially. When the crossover measurements are encountered, the measurement partial derivatives at the first crossover times, $H(t_i)$, are computed and stored on a scratch disk file according to the respective address indices.

Step 3: When the second crossover time, $t_j$, is encountered, the associated address identification index is decoded, and the disk file address for information containing the $t_j$ measurement is calculated. A direct disk access read is then performed on the scratch disk file to obtain $H(t_j)$, from which the crossover partial derivative $H(t_i, t_j)$ and the crossover residual can be computed and accumulated into the information matrix. This procedure is repeated until all measurements are processed. The final result of this step is the information equation containing fully accumulated partial derivatives for all available measurements, including crossover measurements, except for those edited as a result of excessively large residuals or other rejection intervals.

Step 4: The estimate of the state parameter adjustment is obtained using the technique of Gentlemen [20]. If deemed necessary due to large adjustments, the entire process can be repeated beginning with Step 1. Usually, however, the small changes in the orbit produce acceptably small changes in the crossover times and locations from those obtained in the initial execution of Step 1. In this case, another iteration can be performed beginning with Step 2.

The procedure described above has been implemented in UTOPIA, and the application of orbit determination using SEASAT and GEOSAT crossover measurements have been performed.

Sensitivity of Geopotential Coefficients Using Crossover Data

In this section, analytic theory is used to examine the sensitivity of single- and dual-satellite altimeter crossover data to geopotential coefficients. The radial orbit error due to geopotential perturbations can be represented in a spatial domain for near-Earth, near-circular satellite orbits. For a near-circular orbit (orbital eccentricity approximately zero), the radial orbit error can be expressed in terms of the order zero part ($g = 0$ in eccentricity function) as follows (Rosborough [18]):

$$
\Delta r^{(0)} = \sum_{i=1}^{m} \sum_{m=0}^{l} \sum_{\rho=0}^{l} D_{i m \rho} \tilde{\Phi}_{i m \rho}(C_{i m} \cos m\lambda + \tilde{S}_{i m} \sin m\lambda) \\
+ \sum_{i=1}^{m} \sum_{m=0}^{l} \sum_{\rho=0}^{l} D_{i m \rho} \tilde{\Phi}_{i m \rho}(C_{i m} \sin m\lambda - \tilde{S}_{i m} \cos m\lambda)
$$

(5)
where $\Delta r^{(0)}$ is the radial orbit error to order zero in eccentricity ($q = 0$) due to gravity; $D_{lmp}$ is the inclination and altitude function associated with the satellite orbit; $\Phi_{lmp}$ and $\Phi_{lmp}'$ are latitude functions; $\bar{C}_{lm}$ and $\bar{S}_{lm}$ are fully normalized geopotential coefficients of degree $l$ and order $m$; $\lambda$ is the longitude; $lmpq$ are indices for the summation; $+$ sign denotes satellite is on ascending pass; $-$ sign denotes satellite is on descending pass. Note that equation (5) represents only the order zero (in eccentricity) part of the radial perturbation due to gravity. The higher order effects ($q > 0$) for near-circular orbits are negligible for this study.

The first term in equation (5) can be described as the mean radial orbit error due to gravity:

$$\Delta \gamma = \sum_{l=1}^{l} \sum_{m=0}^{m} \sum_{p=0}^{p} D_{lmp} \Phi_{lmp}' \left( \bar{C}_{lm} \cos m \lambda + \bar{S}_{lm} \cos m \lambda \right)$$ (6)

And the second term can be described as the variability error about the mean due to gravity:

$$\Delta \nu = \pm \sum_{l=1}^{l} \sum_{m=0}^{m} \sum_{p=0}^{p} D_{lmp} \phi_{lmp}' \left( \bar{C}_{lm} \sin m \lambda - \bar{S}_{lm} \cos m \lambda \right)$$ (7)

To this level of approximation, for single satellite crossovers, the crossover error, $\Delta x$, can be expressed as the difference of $\Delta r^{(0)}$ (equation (5)) at ascending and descending passes:

$$\Delta x = 2 \Delta \nu$$

$$= 2 \sum_{l=1}^{l} \sum_{m=0}^{m} \sum_{p=0}^{p} D_{lmp} \Phi_{lmp}' \left( \bar{C}_{lm} \sin m \lambda - \bar{S}_{lm} \cos m \lambda \right)$$ (8)

Note that the difference of the mean error, $\Delta \gamma$, (equation (6)) is zero for the same satellite, and zonal harmonic coefficients are thus unobservable to this level of approximation.

For dual-satellite crossovers with distinct inclinations and altitudes, the crossover error between satellites $i$ and $j$ can be defined as follows:

$$\Delta x = \Delta \gamma_i - \Delta \gamma_j + \Delta \nu_i - \Delta \nu_j$$

$$= \sum_{l=1}^{l} \sum_{m=0}^{m} \sum_{p=0}^{p} (D_{lmp} \Phi_{lmp}') \left( \bar{C}_{lm} \cos m \lambda + \bar{S}_{lm} \sin m \lambda \right)$$

$$- \sum_{l=1}^{l} \sum_{m=0}^{m} \sum_{p=0}^{p} (D_{lmp} \Phi_{lmp}') \left( \bar{C}_{lm} \cos m \lambda + \bar{S}_{lm} \sin m \lambda \right)$$

$$\pm \sum_{l=1}^{l} \sum_{m=0}^{m} \sum_{p=0}^{p} (D_{lmp} \Phi_{lmp}') \left( \bar{C}_{lm} \sin m \lambda - \bar{S}_{lm} \cos m \lambda \right)$$

$$\pm \sum_{l=1}^{l} \sum_{m=0}^{m} \sum_{p=0}^{p} (D_{lmp} \Phi_{lmp}') \left( \bar{C}_{lm} \sin m \lambda - \bar{S}_{lm} \cos m \lambda \right)$$ (9)

The sensitivity of geopotential coefficients using altimeter crossover data can thus be inferred from equation (8) and equation (9) for the cases of single- and
dual-satellite crossovers, respectively. For single satellite crossovers, the sensitivity coefficient can be expressed as follows:

\[ S_{n\alpha} = \frac{\Delta \Delta x}{\Delta \alpha \Delta m} \]

\[ = 2 \frac{\partial \Delta \nu}{\partial \alpha \Delta m} \]

(10)

where \( S_{n\alpha} \) is the sensitivity of geopotential coefficient \( \alpha \) of degree \( l \) and order \( m \).

For dual-satellite crossovers, the sensitivity can be expressed as the partial derivative of \( \Delta x \) with respect to the geopotential coefficients:

\[ S_{n\alpha} = \frac{\partial \Delta x}{\partial \alpha \Delta m} - \frac{\partial \Delta \gamma}{\partial \alpha \Delta m} + \frac{\partial \Delta \eta}{\partial \alpha \Delta m} - \frac{\partial \Delta \nu}{\partial \alpha \Delta m} \]

(11)

for satellites \( i \) and \( j \).

Figure 4 shows the sensitivity indices computed for a geopotential field complete to degree and order 36 to the dual-satellite crossovers (TOPEX/ERS-1) using equation (10). To this level of approximation, equation (10) indicates that the sensitivity indices for the zonal harmonics are identically zero for single-satellite crossovers. Zonal harmonics, although weak compared to some geopotential coefficients, are sensitive to the dual-satellite crossover data for TOPEX and ERS-1 orbits (Fig. 4). Figure 5 shows a plot of crossover rms per order of the geopotential predicted for TOPEX, ERS-1 and TOPEX/ERS-1 orbits assuming errors in the geopotential are given by the differences of GEM-10B (Lerch et al. [21]) and GRIM-3B (Reigber et al. [22]) (equations 8 and 9). Figure 5 shows that the TOPEX/ERS-1 dual-satellite crossovers have consistently higher crossover rms over the single-satellite crossovers for TOPEX, meaning that, depending on the orbital characteristics for the two altimetric satellites, the dual-satellite crossovers in general are more sensitive to geopotential coefficients. In particular, order one geopotential coefficients have significantly higher crossover rms in the dual-satellite crossover case than the case for single-satellite crossovers for the TOPEX/ERS-1 orbits.

**Orbit Determination Experiments Using Altimeter Crossover Data**

Gravity field solutions which have included or excluded altimeter crossover data have been evaluated using orbit fits to examine the effect of crossover data to the gravity field solutions. Table 1 shows two 6-day fits for SEASAT (epochs at 1978/7/28 and 1978/9/15) using three different gravity fields. PGS-S4 is the SEASAT-tailored field (Lerch et al. [23]), and the PTGF2 fields are the University of Texas preliminary TOPEX gravity fields (Tapley et al. [17]). One of the PTGF2 fields was obtained with altimeter crossover data excluded from the gravity field solution (Table 1). Crossover residual rms is a good indication of global radial orbit accuracy for altimeter satellite orbits, as the radial orbit error (equation 8) can be approximated by dividing \( \sqrt{2} \) into the crossover residual rms.
FIG. 4. Sensitivity of TOPEX/ERS-1 Dual Satellite Crossovers to Geopotential Coefficients.
Gravity Error Inferred by GEM10B - GRIM3B

$\sim 200$

$\sim 0$

$\sim J.$

10 20 30 40

Geopotential Order


TABLE I. Evaluation of Earth's Gravity Models With and Without Altimeter Crossover Data

- Orbit fits
- SEASAT altimeter crossover data excluded or included in the gravity field solution

<table>
<thead>
<tr>
<th>Crossover Residual $rms$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-day orbit fits</td>
</tr>
<tr>
<td>1978/7/28</td>
</tr>
<tr>
<td>1978/9/15</td>
</tr>
</tbody>
</table>

*GSFC SEASAT-tailored field (Lerch et al.) [23]
$^t$University of Texas preliminary TOPEX gravity fields (Tapley et al. [17])

(Shum [1]). The orbit fits with the PTGF2 field which included Seasat crossover data are at the 40–50 cm level, while the other fields produce fits at meter level. While the crossover $rms$ does not include the geographically correlated (mean) part of the radial orbit error (equation 6), a lower crossover $rms$ indicates the radial orbit error is significantly reduced by inclusion of crossover data in the gravity field solution.

Conclusion

In summary, generalized methodology for the computation of single- and dual-satellite altimeter crossover times, locations and measurements has been devel-
Altimeter Crossover Methods for Precision Orbit Determination

opened. Techniques using single- and dual-satellite crossover measurements for precision orbit determination and reduction of geophysical parameters have been developed. Analytic theory has been used to examine the sensitivity of geopotential coefficients using single- and dual-satellite crossovers. For circular orbits, zonal harmonic coefficients which are insensitive to single-satellite crossovers are sensitive to dual-satellite crossovers. Depending on the orbital characteristics of the altimetric satellite, order one geopotential coefficients using dual-satellite crossovers can be significantly more sensitive than those of single-satellite crossovers. Numerical experiments showed that the use of SEASAT crossover measurements contributes significantly to the accuracy of the Earth's gravity field.

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