Satellite orbit error due to geopotential model error using perturbation theory: applications to ROCSAT-2 and COSMIC missions

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Abstract

A program in FORTRAN 77 is designed to compute satellite orbit error due to geopotential model error. The program is self-contained and is based on an approximate analytical orbital theory that describes the radial, along-track and cross-track positional perturbations of a satellite in a near-circular orbit. Use of the approximate formula reduced programming effort in computing eccentricity functions and the transformation coefficients from Keplerian perturbations to radial, along-track and cross-track perturbations. With this program, the RMS orbit errors due to the EGM96 model error are estimated to be 0.56 m for ROCSAT-2, 64.09 m for COSMIC at a 400-km altitude, and 1.69 m for COSMIC at a 800-km altitude, respectively. The EGM96 model will yield an orbital accuracy consistent with the ground resolution of ROCSAT-2’s optical camera, but will not yield the required sub-meter level orbital accuracy for COSMIC in the operational phase, which is for retrieving meteorological data. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The primary perturbing force acting on a low Earth-orbiting satellite arises from the geopotential; other forces are detailed in, for example, Seeber (1993). Thus, analysis of orbit error due to geopotential model error is important for knowing the achievable accuracy of a given geopotential model in satellite orbit computation. The propagation of geopotential model error to orbit error can be done by a numerical method or an analytical method. In the numerical method, the variational equations (see, for example, Reigber, 1989) first numerically integrated to establish the linear relationship between orbital perturbations and geopotential coefficients. Then, the standard covariance propagation technique (Koch, 1987, p. 116) is used to compute orbit error from the error covariance matrix of a given geopotential model. In the analytical method, the linear relationship between orbital perturbations and geopotential coefficients is obtained by an analytical formula of orbital perturbation such as those given in Wagner (1985), Rosborough and Tapley (1987), Engelis (1987), Schrama (1991), and Hwang (in press). Although the numerical method is more rigorous than the analytical method, the former requires a large programming effort and supporting computer programs (e.g., a program for numerical integration). For a near-circular orbit, defined to be an orbit with eccentricity \( e \leq 0.001 \) in this paper, the analytical method can be simplified considerably and hence requires less programming effort. Furthermore, since in the error propagation some degree of approximation is allowed in computing the design matrix, in most situations an approximate analytical
formula will give a result close to the one from the numerical method (see, for example, Leick, 1995, p. 399).

Since nearly all satellite altimeter missions (e.g., Seasat, Geosat, ERS-1/2, TOPEX/POSEIDON, see Seebier, 1993), the SPOT and LANDSAT series of remote-sensing missions, and the gravity missions CHAMP, GOCE (Balmino et al., 1998) and GRACE (Tapley and Reigber, 2000) have near-circular orbits, an analytical formula of orbital perturbation for near-circular orbit should be applicable to a wide range of satellite missions. With this understanding, the aim of this study is therefore to develop a computer program for orbit error analysis using an analytical formula that is largely based on the results in Rosborough and Tapley (1987) and Hwang (in press). As a case study, this program will be used to predict the orbit errors of ROCSAT-2 and COSMIC satellites due to the error of the EGM96 global geopotential model (Lemoine et al., 1998). ROCSAT-2 is the first remote sensing satellite developed by Taiwan and will be launched in 2003. Its primary payload is an optical camera for monitoring the landmass and ocean around Taiwan in a real-time mode, and a sensor for detecting upper-atmosphere lightning phenomena. ROCSAT-2 will orbit at an altitude of 891 km, and is sun-synchronous with a one-day repeat period. ROCSAT-2’s optical camera has a ground resolution of 2 m. Thus the accuracy of ROCST-2 orbit should be about this order in order not to reduce the capability of the camera in resolving land topography.

COSMIC is a joint Taiwan–US satellite mission that uses a constellation of six satellites to study the atmosphere by the GPS occultation technique (e.g., Rocken et al., 2000). The altitudes of the COSMIC satellites range from 400 km in the transition phase (for geodetic research) to 800 km in the operational phase (for atmospheric research). Accurate orbit calculation is critical to the successful retrieval of meteorological parameters from COSMIC GPS data (Rocken et al., 2000). It is emphasized that the application of our orbit error analysis program will be restricted to near-circular orbits because of the use of an approximate formula.

2. Theory of orbital perturbation

2.1. The earth’s perturbing potential

The geopotential may be represented by a finite series of spherical harmonics as defined by Eq. (1) (Heiskanen and Moritz, 1985, p. 342)

\[ V(r, \phi, \lambda) = \frac{GM}{r} \left[ 1 + \sum_{l=2}^{K} \left( \frac{d e}{a} \right)^l \sum_{m=0}^{l} \left( \hat{C}_{lm} \cos m\lambda + \hat{S}_{lm} \sin m\lambda \right) \hat{P}_{lm}(\sin \phi) \right] = \frac{GM}{r} + R, \]

where \( r, \phi \) and \( \lambda \) are the geocentric spherical coordinates (geocentric distance, geocentric latitude and longitude), \( GM \) is the product of the gravitational constant and the Earth’s mass, \( a_e \) is the semi-major axis of a reference ellipsoid, \( \hat{C}_{lm}, \hat{S}_{lm} \) are fully normalized harmonic coefficients of degree \( l \) and order \( m \), \( K \) is the maximum degree of expansion and \( \hat{P}_{lm} \) is the fully normalized associated Legendre function. In Eq. (1), the geopotential is split into the point-mass potential, \( GM/r \), and the perturbing potential, \( R \). The \( K \) value in Eq. (1) depends on the satellite altitude (Kaula, 1966; Hwang and Lin, 1998); for ROCSAT-2 and COSMIC, \( K = 70 \) is sufficient to account for all currently detectable components of the perturbing geopotential that contribute to satellite perturbation. By a change of variables, \( R \) can be expressed in terms of the six Keplerian elements (Kaula, 1966, p. 37)

\[ R = \frac{GM}{a} \sum_{l=2}^{K} \left( \frac{d e}{a} \right)^l \sum_{m=0}^{l} \sum_{p=0}^{l} \hat{F}_{lm}(l) \sum_{q=-Q}^{Q} \sum_{p=0}^{Q} G_{pq}(e) \sin \theta_{pq}(\omega, M, \Omega, \Theta), \]

where \( a, e \) and \( i \) are, respectively, the semi-major axis, eccentricity, and inclination angle of the osculating orbital ellipse, \( \omega \) is the argument of perige, \( M \) is the mean anomaly, \( \Omega \) is the right ascension of the ascending node, and \( \Theta \) is Greenwich sidereal time (GST). For the definition and geometry of the six Keplerian elements, see, for example, Figs. 2 and 3 in Kaula (1966, pp. 16–17). Furthermore, \( \hat{F}_{lm} \) is the fully normalized inclination function with the same normalizing factor as the fully normalized Legendre function, i.e., \( \hat{F}_{lm} = H_{lm} \hat{F}_{lm} \), with

\[ H_{lm} = \frac{[2 - \delta(m)][2l - 1][l - m]!/[(l + m)!]^2}{2^p(l - p)!}, \]

where \( \delta(m) = 0 \) if \( m \neq 0 \), and \( \delta(m) = 1 \) if \( m = 0 \). In this paper, the following summation formula is used to compute \( \hat{F}_{lm} \) (Emeljanov and Kanter, 1989):

\[ F_{lm}(i) = (-1)^{l}[i(l + m + 1)/2] (l + m)! \times \left[ \frac{\sin i}{\sqrt{2}} \right]^2 \]

\[ \times \left( \frac{\cos i}{\sqrt{2}} \right)^{2l - 2p} \sum_{j=\max(2p - m, 0)}^{\min(2l - 2p, l - m)} \left( \begin{array}{c} 2l - 2p \\ j \end{array} \right) \left( \begin{array}{c} 2p \\ l - m - j \end{array} \right) \]

\[ \times \left( \begin{array}{c} 2l - 2p \\ 2p \\ l - m - j \end{array} \right) \left( \begin{array}{c} 2p \\ l - m - j \end{array} \right), \]

where \( E[(l - m + 1)/2] \) is the integer part of \( (l - m + 1)/2 \) and \( b = m - l + 2p + 2j \). The stability of the summation algorithm in Eq. (4) is tested using a program from Balmino (pers. comm., 1999), who uses a recursive formula similar to the one in Emeljanov and Kanter (1989) to compute inclination functions and their derivatives. Such a recursive formula is numerically stable up to at least degree 100,000 (Emeljanov and
Kanter, 1989, p. 83). The relative error by degree of the summation algorithm is defined as
\[
e_l = \sum_{m=0}^{l} \frac{\sum_{n=0}^{l} \varepsilon_{lmp}}{\sqrt{(l+1)^2}} \quad \text{with} \quad \varepsilon_{lmp} = \left| \frac{F_{lmp}^r - F_{lmp}^t}{F_{lmp}^r} \right|,
\]
where \(F_{lmp}^r\) and \(F_{lmp}^t\) are inclination functions (or derivatives) computed with the summation algorithm and the recursive algorithm, respectively. The test result shows that \(e_l \approx 10^{-6}\) for \(l < 62\), and is about \(10^{-4}\) for \(63 < l \leq 70\). Thus the summation algorithm in Eq. (4) is numerically stable up to degree 70. If one wishes to compute the inclination function for \(l > 70\), one can use the recursive formula to avoid numerical instability. Furthermore, \(G_{lmp}(e)\) is the eccentricity function, and \(Q\) is the limit of \(q\) and depends on the eccentricity \((G_{lmp}(e)\) is of the order of \(e^{-|l|}\), so, for example, for \(e < 0.001\) \(Q\) may be 1). Finally,
\[
S_{lmpq} = \left[ \begin{array}{c} C_{lm}^+ \\ -S_{lm}^- \end{array} \right] \sin \psi_{lmpq} + \left[ \begin{array}{c} S_{lm}^+ \\ -C_{lm}^- \end{array} \right] \cos \psi_{lmpq},
\]
where \(C_{lm}^+\) is \(C_{lm}\) when \((l - m)\) is even, and \(C_{lm}^-\) is \(C_{lm}\) when \((l - m)\) is odd, and
\[
\psi_{lmpq} = (l - 2p)\omega + (l - 2p + q)\Omega + m(\Omega - \Theta).
\]

2.2. Lagrange’s equations of motion and perturbations of Keplerian elements

To obtain orbital perturbations in any desired directions, one can start with the perturbations in the six Keplerian elements (coordinates). First, in a geocentric Cartesian coordinate system, the equations of motion (EOM) of an Earth-orbiting satellite are three second-order ordinary differential equations (ODE) with the time as the independent variable (Kaula, 1966). In order to obtain an analytical solution of EOM, the EOM must be expressed in a non-rectangular coordinate system. The use of Keplerian elements in the EOM results in six first-order ODEs, often called Lagrange’s equations of motion (LEOM) (Kaula, 1966, p. 29):
\[
\begin{align*}
\frac{da}{dt} &= \frac{2}{na} \frac{\partial R}{\partial M}, \\
\frac{de}{dt} &= 1 - e^2 \frac{\partial R}{\partial M} - e \sin i \frac{\partial R}{\partial \omega}, \\
\frac{di}{dt} &= \cos i \frac{\partial R}{\partial M} - \sin i \frac{\partial R}{\partial \Omega}, \\
\frac{d\omega}{dt} &= -\frac{\sin i}{\sqrt{1 - e^2 \sin^2 i}} \frac{\partial R}{\partial i}, \\
\frac{d\Omega}{dt} &= \frac{1}{\sqrt{1 - e^2 \sin^2 i}} \frac{\partial R}{\partial i}, \\
\frac{dM}{dt} &= n - 1 - e^2 \frac{\partial R}{\partial e} - 2 \frac{\partial R}{\partial a},
\end{align*}
\]
where \(n = \sqrt{GM/a^3}\) and \(R\) is the perturbing potential defined in Eq. (2). For a near-circular orbit, Kaula (1966) gives an analytical, approximate solution of Eq. (8). Kaula’s solution requires a reference orbit whose semi-major axis, eccentricity and inclination (denoted as \(\bar{a}, \bar{e}\) and \(\bar{i}\)) remain near-constant, and argument of perigee, right ascension of ascending node and mean anomaly undergo a linear variation with time such that
\[
\begin{align*}
\omega(t) &= \omega_0 + \dot{\omega}(t - t_0), \\
\Omega(t) &= \Omega_0 + \dot{\Omega}(t - t_0), \\
M(t) &= M_0 + \dot{M}(t - t_0),
\end{align*}
\]
where \(\omega_0, e_0, M_0\) are the mean elements at a reference epoch \(t_0\), and \(\dot{\omega}, \dot{\Omega}, \dot{M}\) are their linear rates of change with time \(t\). The nine orbital parameters \((\bar{a}, \bar{e}, \bar{i}, \omega_0, \Omega_0, M_0, \dot{\omega}, \dot{\Omega}, \dot{M})\) and the reference epoch \(t_0\) then describe a reference orbit. With the reference orbit, both sides of Eq. (8) may be integrated with respect to time to yield the Keplerian perturbations:
\[
\Delta x = \sum_{l=2}^k \sum_{m=0}^l \sum_{p=0}^l \sum_{q=-\infty}^\infty \Delta x_{lmpq},
\]
where symbol \(x\) represents any of the six Keplerian elements. For perturbations in \(a, e, i, \omega, \Omega, M\), we have
\[
\Delta x_{lmpq} = C_{lmpq}^x S_{lmpq},
\]
where
\[
S_{lmpq} = \left[ \begin{array}{c} C_{lm}^+ \\ -S_{lm}^- \end{array} \right] \sin \psi_{lmpq} + \left[ \begin{array}{c} S_{lm}^+ \\ -C_{lm}^- \end{array} \right] \cos \psi_{lmpq},
\]
Expressions for the coefficients, \(C_{lmpq}^x\), are (Kaula, 1966, p. 40)
\[
C_{lmpq}^x = 2\bar{a} \left( \frac{d_x}{\bar{a}} \right)^t \bar{F}_{lmpq} G_{lmpq} \left( \frac{1 - e^2}{\bar{e}} \right)^{1/2} \frac{n}{\psi_{lmpq}},
\]
\[
C_{lmpq}^x = \left( \frac{d_x}{\bar{a}} \right)^t \left[ (1 - e^2)^{1/2} \bar{F}_{lmpq} G_{lmpq} \right] \left( \frac{1 - e^2}{\bar{e}} \right)^{1/2} \frac{n}{\psi_{lmpq}},
\]
\[
C_{lmpq}^x = \left( \frac{d_x}{\bar{a}} \right)^t \left[ (1 - e^2)^{1/2} \bar{F}_{lmpq} G_{lmpq} \right] \left( \frac{1 - e^2}{\bar{e}} \right)^{1/2} \frac{n}{\psi_{lmpq}},
\]
and so on.
where
\[ \psi_{impq} = (l - 2p)\dot{\omega} + (l - 2p + q)\dot{M} + m(\dot{\theta} - \Theta) \] (15)
is the frequency of the perturbations, \( G'_{pq} = dG_{pq}/de \), \( \dot{F}_{imp} = d\dot{F}_{imp}/di \) and \( \Theta \) is the Earth’s mean rotational rate.

If the satellite ephemeris is given, \( \dot{a}, \dot{e}, \text{ and } \dot{l} \) can be obtained by averaging \( a, e \) and \( l \), and \( \omega_0, \Omega_0, M_0, \dot{\omega}, \dot{\Omega}, \text{ and } \dot{M} \) by the linear expressions in Eq. (9). Since there is no predicted ephemeris for the ROCSAT-2 and COSMIC missions, in this paper the angular velocities \( \dot{\omega}, \dot{\Omega}, \dot{M} \) are simply computed from the \( C_{20} \) perturbations as
\[
\dot{\omega} = \frac{d\omega}{dr} = \frac{3nC_{20}a^2}{4(1 - e^2)a^2}(1 - 5\cos^2\tilde{\tau}),
\]
\[
\dot{\Omega} = \frac{d\Omega}{dr} = \frac{3nC_{20}a^2}{2(1 - e^2)a^2}\cos \tilde{\tau},
\]
\[
\dot{M} = \frac{dM}{dr} = n - \frac{3nC_{20}a^2}{4(1 - e^2)a^2}(3 \cos \tilde{\tau} - 1).
\] (16)

Table 1 compares the numerically determined (by least-squares fitting) and analytically determined (by Eq. (16)) velocities using a numerically integrated orbit of COSMIC at an 800-km latitude; see Hwang (in press). These two sets of velocities agree well, and such a result is consistent with the result given in Engelis (1987, p. 45).

A program (reorf) in FORTRAN 77 for numerically determining a reference orbit from a given ephemeris is also available from the authors.

2.3. Radial, along-track and cross-track perturbations

The Keplerian perturbations can be transformed to the radial, along-track and cross-track perturbations. Rigorous formulae for such a transformation have been derived by many authors, e.g., Rosborough and Tapley (1987), Schrama (1991) and Hwang (in press). Expressing orbital perturbations in these three directions allows for assessment of orbital errors in the vertical (radial) and horizontal (along-track and cross-track) components of the orbit. Although there exist rigorous formulae, for a near-circular orbit it is sufficient to use an approximate formula for orbit error computation. One frequently used approximate formula is the so-called order-zero formula in which the radial, along-track and cross-track perturbations are expressed in Keplerian perturbations as (Rosborough and Tapley, 1987; Hwang, in press)
\[
\Delta x_1^0 = (1 - \dot{e} \cos M)\Delta a - (\dot{a} \cos M)\Delta e + (\ddot{a} \sin M)\Delta M,
\]
\[
\Delta x_2^0 = \dot{a}[(\Delta \omega + \Delta M + (\cos \tilde{\tau})\Delta \Omega),
\]
\[
\Delta x_3^0 = \Delta a[(\sin(\omega + M))\Delta \omega - (\sin \tilde{\tau} \cos(\omega + M))\Delta \Omega].
\] (17)

For a near-circular orbit it is sufficient to retain only the \( q = -1, 0, 1 \) terms in the eccentricity function \( G_{pq} \), which may be approximated by (Balmino, 1994)
\[
G_{ip0} = 1,
\]
\[
G_{ip1} = \frac{e}{2}(3l - 4p + 1),
\]
\[
G_{ip(-1)} = \frac{e}{2}(l + 4p + 1). \] (18)

Substituting Eqs. (10), (14) and (18) into Eq. (17), with the function–product relations in trigonometry such as \( \sin z \sin \beta = \frac{1}{2}[\cos(z - \beta) - \cos(z + \beta)] \) it can be shown that (Rosborough and Tapley, 1987; Hwang, in press)
\[
\Delta x_1^0 = \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{n=0}^{l} C_{lm}^1 S_{lm}^0,
\]
\[
\Delta x_2^0 = \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{n=0}^{l} C_{lm}^2 S_{lm}^0,
\]
\[
\Delta x_3^0 = \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{n=0}^{l} (C_{lm}^3 S_{lm}^0 + C_{lm}^3 S_{lm}^0) = C_{lm}^3 S_{lm}^0 (l = 1, m = 0) \] (19)

where
\[
C_{lm}^1 = \frac{na}{a} \hat{F}_{imp} \left[ \frac{2(l - 2p)}{\psi_{imp0} + 4p - 3l - 1} + \frac{4p - l + 1}{2\psi_{imp1}} \right],
\]
\[
C_{lm}^2 = \frac{na}{a} \hat{F}_{imp} \left[ \frac{2(l + 1) - 3(l - 2p)n}{\psi_{imp0}} + \frac{4p - 3l - 1}{\psi_{imp1}} + \frac{l - 4p - 1}{\psi_{imp(-1)}} \right],
\]
\[
C_{lm}^3 = \frac{na}{a} \frac{n}{\psi_{imp0}} \left[ \frac{(l - 2p)\sin \tilde{\tau} - m\hat{F}_{imp}}{\sin \tilde{\tau} \hat{F}_{imp}} \right]. \] (20)

Table 1
Comparison of mean angular velocities of COSMIC at 800-km altitude from analytical and numerical methods

<table>
<thead>
<tr>
<th>Linear rate</th>
<th>Analytical (degree/day)</th>
<th>Numerical (degree/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{\Omega} )</td>
<td>-1.148156</td>
<td>-1.145177</td>
</tr>
<tr>
<td>( \dot{\omega} )</td>
<td>-2.80748</td>
<td>-2.80748</td>
</tr>
<tr>
<td>( \dot{M} )</td>
<td>5143.71178</td>
<td>5143.72433</td>
</tr>
</tbody>
</table>
and
\[
S_{(\pm 1)\mu p0}^* = \left[ \tilde{C}_{lm}^T - \bar{S}_{lm} \right] \sin \psi_{(\pm 1)\mu p0} = \left[ \bar{S}_{lm} \right] \cos \psi_{(\pm 1)\mu p0}.
\]

(21)

For an orbit altitude of about 1000 km, Eq. (19) is valid for about one week (Hwang, 2001). A new reference orbit must be used for subsequent arc segments. The order-zero formula is a good approximation to the exact formula, especially for the example of a near-circular orbit. According to the test result in Hwang (in press, Table 1), the relative accuracy of the order-zero formula is about 1/100. So for orbit error of the order of 1 m, the uncertainty introduced by the formula itself is about 1 cm. With the order-zero formula there will be no need to compute eccentricity functions and to compute the transformation coefficients from Keplerian perturbations to radial, along-track and cross-track perturbations, hence programming effort can be reduced considerably as compared to using a rigorous analytical formula.

3. Description of program orberr

Eq. (19) describes a linear relationship between positional perturbations of a satellite orbit with near-circular eccentricity and geopotential coefficients. In a matrix notation, Eq. (19) can be expressed as
\[
P = \begin{bmatrix}
\Delta x_1^0 \\
\Delta x_2^0 \\
\Delta x_3^0
\end{bmatrix} = GC,
\]
where \( G \) is the design matrix and \( C \) is a vector containing geopotential coefficients. Given the full error covariance of \( C \), \( \Sigma_C \), the error covariance of the radial, along-track and cross-track positional components, \( \Sigma_p \), can be computed as
\[
\Sigma_p = G \Sigma_C G^T,
\]
where \( G^T \) is the transpose of \( G \). The diagonal elements of \( \Sigma_p \) are the error variances of the positional components and can be computed efficiently by the following summation without using matrix products:
\[
\sigma_k^2 = \sum_{i=1}^{n} a_{ki}^2 s_{ij}^2 + 2 \sum_{i=1}^{n} \sum_{j+i=1}^{n} a_{ki} a_{kj} s_{ij} \quad \text{for } k = 1, 2, 3, \quad (24)
\]
where \( k = 1, 2, 3 \) represent the radial, along-track and cross-track directions, respectively, \( s_{ij} \) is the element of \( \Sigma_C \) at the \( i \)th row and \( j \)th column, \( a_{ki} \) is the element of \( G \), and \( n \) is the number of harmonic coefficients, which is \((K+1)^2 - 4\) \((K \) is the maximum degree of expansion, see Eq. (1)). Based on the order-zero formulae in Eq. (19) and error propagation in Eq. (23), we design a program, called orberr, in FORTRAN 77, to compute radial, along-track and cross-track orbit errors due to geopotential model error. The usage of orberr is

orberr -Ccov_mat -Error -Idelt -LK
-Orbit-Tstart/stop,

where \( C \) is the binary file of covariance matrix of geopotential coefficients (input), \( E \) the file of orbit errors, containing longitude, latitude radial, along-track and cross-track errors in meters, \( I \) the increment interval of time (in second) in the output and \( L \) the maximum degree of geopotential coefficients, \( O \) the file of mean orbit elements: \( t_0, \tilde{a}, \tilde{e}, \tilde{I}, \omega_0, M_0 \), with \( t_0 \) being the reference epoch of the elements in modified Julian day (MJD) (for the definition of MJD, see, e.g., Seeber, 1993) and \( T \) the start/stop times of the output trajectory in MJD.

A program (egm96t2b) is also designed to convert a covariance matrix from the NASA/GEODYN format (McCarthy et al., 1993) to the required binary format for option–C in orberr. In orberr the mean angular velocities \( \hat{\omega}, \hat{\Omega}, \hat{M} \) are automatically computed using Eq. (16). Again, as stated in Section 2.3, program reobf can be used to compute the mean Keplerian elements and \( \hat{\omega}, \hat{\Omega}, \hat{M} \) if the satellite ephemeris is given. For option–E, the geodetic coordinates (latitude and longitude) are computed from Keplerian elements by considering only the effect of Earth rotation. For related theories on coordinate transformation in satellite

Table 2
Mean orbital elements of ROCSAT-2 and COSMIC satellites used for computing orbit errors

<table>
<thead>
<tr>
<th>Mean element</th>
<th>Notation</th>
<th>ROCSAT-2</th>
<th>COSMIC (400 km)</th>
<th>COSMIC (800 km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference epoch</td>
<td>( t_0 ) (MJD)</td>
<td>50591.500694</td>
<td>50591.500694</td>
<td>50591.500694</td>
</tr>
<tr>
<td>Semi-major axis</td>
<td>( \tilde{a} ) (km)</td>
<td>7262</td>
<td>6778</td>
<td>7178</td>
</tr>
<tr>
<td>Eccentricity angle</td>
<td>( \tilde{e} )</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Inclination angle</td>
<td>( \tilde{I} ) (°)</td>
<td>98.99</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>Right asc. of nodes</td>
<td>( \Omega_0 ) (°)</td>
<td>110</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Arg. of perigee</td>
<td>( \omega_0 ) (°)</td>
<td>40</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Mean anomaly</td>
<td>( M_0 ) (°)</td>
<td>20</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>
geodesy, see, e.g., Seeber (1993, Chapter 2). In addition, subroutine “kep2geo” in program orberr computes geodetic coordinates from Keplerian elements, and subroutine “rec2kep” in program reforb computes Keplerian elements from inertial rectangular coordinates. The osculating Keplerian elements as determined from the reference orbit are regarded as the nominal orbit of the satellite. The output contains geodetic coordinates and the three orbit error components, allowing users to see the geographic pattern of the orbit error of a satellite. In Eq. (20), when the frequency is zero, the perturbation formulae become singular. In fact, when \( \psi_{impq} \) approaches zero, the resulting perturbations become excessively large. These are the situations of perfect resonance (\( \psi_{impq} = 0 \)) and near-perfect resonance (\( \psi_{impq} \approx 0 \)). In practice, if \( |\psi_{impq}|/M < 0.01 \), program orberr simply sets \( c_{imp} = c_{imp} = c_{imp} = c_{imp} = 0 \) for \( q = -1, 0, 1 \). The cutoff frequency of 0.01 \( M \) has been the optimum choice in that the “approximate” orbital perturbations computed from the analytical formula in Eq. (19) agree best with “true” orbital perturbations computed from numerical integrations (Hwang, in press). Using this cutoff frequency yields a relative error of less than 1% (Hwang, in press).

4. Case study: orbit errors of ROCSAT-2 and COSMIC due to the EGM96 model error

As a case study, the error covariance matrix of the EGM96 model complete to degree and order 70 is used to compute the orbit error of ROCSAT-2 in a one-day arc (ROCSAT-2 has a repeat period of one day) and the orbit error of COSMIC in a three-day arc. The mean elements of ROCSAT-2 and COSMIC used for generating the orbit errors are given in Table 2 (some of the elements are based on Rocken et al., 2000). Although the radial, along-track and cross-track orbit error components as computed from Eq. (23) are mutually correlated, the concept of dilution of position (DOP) in Global Positioning System (GPS) (e.g., Leick, 1995, p. 253) can be used to define the horizontal and total orbit errors without considering the correlations. In other words, the horizontal orbit error can be computed by

\[
\sigma_h = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2},
\]

and the total orbit error by

\[
\sigma_T = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2},
\]

where \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are defined in Eq. (24). Fig. 1 shows the radial, along-track and cross-track errors as functions of time along these satellite arcs. Figs. 2–4 show the geographic patterns of the orbit errors and satellite...
Fig. 2. Geographic patterns of radial (top), horizontal (middle) and total orbit errors (in meters) of ROCSAT-2 due to EGM96 model error to degree 70.
Fig. 3. Geographic patterns of radial (top), horizontal (middle) and total orbit errors (in meters) of COSMIC due to EGM96 model error to degree 70. Units are meters. Satellite altitude is 400 km.
Fig. 4. Geographic patterns of radial (top), horizontal (middle) and total orbit errors (in meters) of COSMIC due to EGM96 model error to degree 70. Units are meters. Satellite altitude is 800 km.
ground tracks. Tables 3 and 4 show the statistics of the orbit errors. A summary of the error characteristics in these figures and tables is as follows:

1. The along-track error is the largest, than the radial error and the cross-track error is the smallest. The magnitude of the error depends on satellite altitude; the lower the altitude, the larger the error, which is due to the attenuating factor \( \frac{\Delta e}{r} \) in Eq. (1).

2. For the same satellite orbit, the radial error is the smallest near the equator, and increases with latitude.

3. The horizontal error is composed of the along-track and cross-track errors, thus the horizontal error is larger than the vertical error.

4. The orbit error on land is larger than that over the oceans. The largest error occurs near the Asian continent. This is probably because of lack of good terrestrial gravity data in computing the EGM96 model there. Using the satellite-only model of EGM96, EGM96S (Lemoine et al., 1998), may improve the orbit accuracy on land.

5. Conclusions

In this paper, the orbit perturbation theory of Kaula (1966) was used to derive an approximate analytical formula for near circular orbit—the order-zero formula, suitable for assessing orbit errors in the radial, along-track and cross-track directions. The use of the order-zero formula has reduced programming effort considerably, mainly because of no need to compute eccentricity functions and the transformation coefficients from keplerian perturbations to positional perturbations. With the order-zero formula a program (orberr) was designed to compute radial, along-track and cross-track satellite orbit errors due to geopotential model error. The program is self-contained, requiring no supporting libraries or subroutines. As a case study, this program is applied to the predictions of orbit errors of ROCSAT-2 and COSMIC satellites due to the EGM96 model error up to degree 70. The RMS orbit errors due to the EGM96 model error are estimated to be 0.56 m for ROCSAT-2, 64.09 m for COSMIC at a 400-km altitude, and 1.69 m for COSMIC at a 800-km altitude, respectively. From our analysis, the EGM96 model will fulfill the accuracy requirement of ROCSAT-2 provided that other perturbing forces are accurately modeled. The EMG96 model will not yield the required sub-meter level orbit accuracy for COSMIC in the operational phase, which is for retrieving meteorological data. However, at the time (2004) when the COSMIC satellites are launched an improved geopotential model (over EGM96) should be available to fulfill the accuracy requirement of COSMIC orbit determination. All the related programs and test data sets are available at the WWW site: http://space.cv.nctu.edu.tw/Research/orbit_error.html, or from the IAMG server.

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<th>Table 3</th>
<th>Statistics of orbit error (in meters) of ROCSAT-2 satellite due to EGM96 model error to degree 70a</th>
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<td></td>
<td>Radial error</td>
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<td>Maximum</td>
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<td>Minimum</td>
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<tr>
<td>Mean</td>
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<tr>
<td>RMS</td>
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aRMS: root mean squared error.

<table>
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<th>Table 4</th>
<th>Statistics of orbit error (in meters) of COSMIC satellite due to EGM96 model error to degree 70</th>
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<tr>
<td></td>
<td>Altitude 400 km</td>
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<tr>
<td></td>
<td>Radial</td>
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<td>Maximum</td>
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<td>RMS</td>
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two anonymous reviewers greatly improved the quality of this paper.

References


