CHAPTER 1

INTRODUCTION

1.1 Satellite Radar Altimetry

Satellite radar altimetry is a revolutionary technology to measure the height and the shape of the sea surface globally from space. It has been used to observe global oceanic topography and its changes with unprecedented accuracy (several cm root-mean-square (RMS) in sea surface heights (SSH)) and resolutions (up to 50 km spatial scale and weekly temporal sampling) (Chelton et al., 2001; Shum et al., 2003). The observation principle of satellite altimetry is conceptually straightforward. It transmits an electromagnetic pulse to the sea surface and measures its two-way travel time when the return reflected from the instantaneous sea surface is received. The altimeter-observed time delay \( t \) can be converted to the range \( R \) from the satellite to the ocean surface as:

\[
R = \frac{ct}{2}
\]  

where \( c \) is the free-space speed of light.

This instantaneous range measurement must be corrected for the instrument (e.g., oscillator drift, Doppler shift, range acceleration), the atmospheric propagation (e.g., wet and dry troposphere, ionosphere), and geophysical surface height variations (e.g., ocean tide, solid Earth tide, pole tide). With a precise orbital height \( H \) from the satellite to a reference ellipsoid, a sea surface height \( h_{ssh} \) relative to the specific reference ellipsoid can be computed as:

\[
h_{ssh} = H - h_{alt}
\]  

where \( h_{alt} \) is the range after all of the corrections applied.

The altimeter measures the range using an on-board tracker which provides a time series of the received power distribution of the reflected pulses, which is called the

Extracted from Lee, H., Radar altimetry methods for solid earth geodynamics studies, PhD Dissertation, Ohio State University [2008]
altimeter waveform. Figure 1.1 shows a schematic description of a pulse-limited radar altimeter obtained from a diffuse, horizontal and planar sea surface. The pulse-limited radar altimeter transmits a spherically expanding pulse of duration $\tau$ (for TOPEX/POSEIDON, 3.125 nsec), and thus the pulse width is $c\tau$, where $c$ is the speed of light. When the leading edge of the pulse arrives at the sea surface at nadir ($t = t_0$), the spherically expanding pulse starts to illuminate the sea surface at nadir, and the reflected signal begins to return to the altimeter. At a two-way travel time $t_0 < t < t_1$, the leading edge of the pulse continues to illuminate the planar sea surface further from the nadir point. Therefore, the illuminated area, which contributes to the radar return becomes an

![Figure 1.1: Interaction of a pulse and the scattering surface, and the developed returned waveform. PLF is the pulse-limited footprint, and $\theta_a$ is the antenna beam width [after Deng (2003)].](image)
expanding circle until the trailing edge of the pulse reaches the sea surface \( (t = t_1) \). At this time, the backscattered energy to the altimeter reaches its maximum. Thereafter \( (t > t_1) \), the footprint (or the illuminated area) becomes an annulus and the returned signal at the altimeter starts to decay due to the finite antenna beam width. Therefore, typical ocean waveforms have a rapidly rising leading edge and a slowly decaying trailing edge.

**Figure 1.2**: Example of TOPEX averaged return waveform over deep ocean.

Figure 1.2 shows an example of a TOPEX 64-sample waveform obtained over deep oceans. This waveform is obtained by averaging 480 successive returned pulses over a “track interval” spanning about 100 msec, which correspond to the interval of ten-per-frame (or 10 Hz) range measurements to reduce the “Rayleigh fading” of the radar signal (Chelton et al., 2001). The waveform obtained on-board has 128 samples, uniformly spaced at 3.125 nsec, which is combined (averaged) into the 64 samples and placed into the telemetry to the ground (Hayne et al., 1994).

The two-way travel time is defined as:

\[
t_{1/2} \equiv \frac{t_0 + t_1}{2} = \frac{2R}{c} + \frac{\tau}{2}
\]

(1.3)
The two-way travel time thus represents the time for the midpoint of the pulse to return from sea surface at nadir. Therefore, the altimeter determines the two-way travel time by identifying the half-power point on the leading edge of the waveform. Due to the inherent noise in the waveform as shown above, the on-board tracker is designed, based on a Brown (1977) model of the return waveform, to align the return spectral waveform so that the half-power point of the leading edge is at a specified frequency $f_0$. This tracking point corresponds to the gate 32.5 for 128-sample waveform or 24.5 for 64-sample waveform of TOPEX. The on-board tracker determines the offset between the half-power point on the leading edge of the waveform and the tracking point and realigns subsequent waveforms with the half-power point at the frequency $f_0$ (Chelton et al., 2001). This frequency shift corresponds to the two-way travel time from the altimeter to mean sea surface at satellite nadir.

The Brown model (1977) is the physical basis of the ocean waveform which is represented by the convolution of the instrument point target response (PTR), the flat surface response (FSR) and the probability density function of the specular points (particles on the surface which reflects the radar signal) (Brown, 1977; Hayne, 1980; Rodriguez, 1988) (see Chapter 2.3.1 for more details). Under the assumption that the distribution of the specular points is Gaussian, two-way travel time corresponds to the half-power point of the leading edge. However, this assumption is a primary limitation of the on-board tracker because the actual distribution of the specular points is skewed Gaussian. These effects may be corrected for in ground-based post processing, either by analyzing the waveform data, or by use of the look-up table correction (Chelton et al., 1989).

1.2 Applications of Altimeter Measurements over Non-Ocean Surfaces

Although satellite radar altimetry was initially designed for oceanographic applications such as sea level change, ocean circulation, and ocean tides, it has been demonstrated to be applicable to non-ocean surfaces as well. However, the performance of the radar altimeter over varying terrain differs significantly from that over ocean surface, and may lead the altimeter’s on-board tracker to fail in precisely predicting the range. Therefore, the altimeter range measurements over non-ocean surfaces must be corrected for the deviation of the mid-point of the leading edge from the tracking gate of the onboard tracker (for TOPEX 64-sample waveform, 24.5). This procedure is known as re-tracking. It has been shown that radar altimetry is capable of measuring surface elevations and changes over ice sheets, including Antarctica and Greenland, and subsequently the ice sheet mass balance and its role in global sea level change (Zwally et al., 1989; Wingham et al., 1998; Davis et al., 2005). Furthermore, through the analysis of radar waveforms, the profile of backscattered power and the geophysical information related to the near-surface properties of the ice sheet such as the extinction coefficient (power lost from the incident energy due to scattering and absorption) can be derived.
(Davis and Zwally, 1993). In addition, satellite radar altimetry has been used to measure inland water level variation for hydrologic studies (Birkett, 1998; Birkett et al., 2002; Frappart et al., 2006). Whereas these studies focused on large river basins such as the Amazon, attempts have been made to measure water level change over vegetated wetlands using TOPEX radar altimetry. Several studies (Brooks et al., 1997; Deng et al., 2002; Deng and Featherstone, 2006; Hwang et al., 2006) have also been conducted to improve the SSH estimates in coastal regions where altimetry-derived SSH are likely in error due to the complex nature of the return waveform from rapidly varying coastal topographic surfaces (Deng, 2003).
CHAPTER 2

SATELLITE RADAR ALTIMETER DATA PROCESSING

2.1 TOPEX/POSEIDON and Envisat Altimeter Data

TOPEX/POSEIDON (T/P) carries the first dual-frequency (Ku- and C-band) radar altimeter space mission designed to accurately measure global ocean topography with unprecedented accuracy. T/P satellite was launched on August 10, 1992, on a near-circular orbit repeating every 9.9 days. The orbit inclination is 66° that enables the global observation of the ocean within 66° latitude bounds. The POSEIDON altimeter, which shared the antenna on-board the satellite and was operated for about 10 percent of the mission time, is not used in this study. The TOPEX altimeter has redundant Sides A and B hardware. The signals from Side A altimeter began to show performance degradation in 1999, and it has been switched to Side B on 10 February 1999 (Hayne and Hancock, 2000). In December 2001, T/P and Jason-1 were placed in the same orbit forming a so-called tandem phase where they are separated by only about 70 seconds. The tandem phase lasted about 7 months and T/P has been moved to the orbit with the ground tracks in between its old ones on August 2002.

In this study, TOPEX GDRs and Sensor Data Records (SDRs) for cycle 9-364 and Envisat were used. The TOPEX GDRs and SDRs are available from the NASA/JPL Physical Oceanography Distributed Active Archive Center (PO.DAAC). The Ku-band 10-Hz range data contained in GDRs are utilized, and their 10-Hz geodetic coordinates per each cycle can be computed from the Precise Orbit Ephemeris (POE) data provided by NASA Goddard Space Flight Center (GSFC) (N. Zelensky, personal communication) using the 10-Hz time tags calculated from the 20-Hz range measurements contained in SDRs (see Appendix A). Detailed description of the SDR data records can be found in Algiers et al. (1993). The SDRs also contain 10-Hz 64-bin waveform measurements, which are used for retracking. The waveform anomalies such as zero-leakage and the offset leakage effects are mitigated by employing the sets of multiplicative and additive waveform factors (Hayne et al., 1994). These factors are originally developed for the TOPEX Side A altimeter, but also applicable to Side B (D. Hancock, personal communication, 2006). Geophysical corrections such as dry troposphere, solid Earth tide
and pole tide corrections have been applied using the values provided in the GDR. Although T/P is equipped with a three-channel (18, 23, and 37 GHz) microwave radiometer for measuring integrated water vapor contents, interpolated wet troposphere correction based on the European Center for Medium Range Weather Forecasting (ECMWF) atmospheric model is used in this study since the wet radiometric observations at these frequencies are overwhelmed by the opaqueness of the land surface, rendering them not usable. The ionosphere correction based on DORIS data is used instead of the dual-frequency ionosphere correction which is based on observing the range differences between the Ku- and C-band channels.

The Envisat RA-2 altimeter is designed to provide a global scale collection of radar echoes over ocean, land, and ice to measure ocean topography, water level variations over the large river basins, land surface elevation and to monitor sea ice and polar ice caps (Wehr and Attema, 2001; Frappart et al., 2006). RA-2 is also a dual-frequency radar altimeter: 13.575 GHz in Ku-band and 3.2 GHz in S-band. Specifically, four different retrackers are operationally applied to RA-2 raw data to provide accurate height estimates, which are included in Envisat RA-2 GDR. Each retracker has been developed for a specific radar return: one for ocean (OCEAN), two for ice sheets (ICE-1 and ICE-2), and one for sea ice (SEA ICE).

2.2 Convolution Models for Ocean Waveforms

The radar altimeter waveforms are the basic measurement for observing the Earth’s surfaces. They provide the range between the satellite and the surface at nadir via two-way travel time of the transmitted pulse, the Significant Wave Height (SWH) via the slope of the waveform leading edge, and the backscattering coefficients which represent the surface roughness and characteristics via the returned power. The shape of the waveform from incoherent surface scattering has been based on physical optics theory, which treats the surface as a set of specular points with a given height and slope probability density distribution. The time series of the mean returned power, i.e., the waveform, \( P(t) \), is represented by the convolution of the instrument point target response (PTR) \( \chi(t) \), the impulse response from a smooth sphere (the mean Earth) \( S(t) \), and the probability density function (PDF) of the specular points \( f_{sp}(t) \) (Brown, 1977; Hayne, 1980; Barrick and Lipa, 1985; Rodriguez, 1988):

\[
P(t) = S(t) \otimes \chi(t) \otimes f_{sp}(t)
\]  \hspace{1cm} (2.1)
The time $t$ is the time measured at the satellite such that $t = 0$ corresponds to the first arrival time of an impulse from the mean ocean surface. To provide a simplified analytical expression, Brown (1977) proceeded with the assumptions that are inherent in the convolution model of near normal incidence rough surface backscatter. These assumptions are generally satisfied over ocean surface, but not for land scatter.

The radar impulse response from a smooth sphere is given by (Hayne, 1980; Barrick and Lipa, 1985; Rodriguez, 1988):

$$S(t) = A \exp(-\alpha t) I_0(\beta t^{1/2}) U(t)$$

$$\alpha = \frac{\ln 4}{\sin^2(\theta/2)} \frac{c}{h} \frac{1}{1 + h/R} \cos(2\xi)$$

$$\beta = \frac{\ln 4}{\sin^2(\theta/2)} \left( \frac{c}{h} \frac{1}{1 + h/R} \right)^{1/2} \sin(2\xi)$$

(2.2)

with:

- $A$ : scaling constant;
- $I_0$ : modified Bessel function of the second kind;
- $U(t)$ : unit step function;
- $h$ : altimeter height above the mean ocean surface;
- $R$ : radius of the Earth;
- $c$ : speed of light;
- $\theta$ : antenna half-power beamwidth ($\approx 0.38^\circ$ for T/P);
- $\xi$ : off-nadir pointing angle.

The derivation of Equation (2.2) assumes that the antenna gain can be approximated by a Gaussian and includes effects due to the Earth’s curvature.

The specular point PDF in spatial domain can be expressed as (Rodriguez, 1988):
\[ f_{sp}(z) = \frac{1}{(2\pi)^{1/2}} \sigma \exp(-\eta^2/2) \left[ 1 + \frac{\lambda}{6}(\eta^3 - 3\eta) \right] \]

(2.3)

\[ \eta = \frac{z - z_r}{\sigma} \]

with:

Equation (2.3) continued

- \( \sigma \): ocean surface standard deviation;
- \( \lambda \): ocean surface skewness;
- \( z \): height above mean ocean surface \((z = 0)\);
- \( z_r \): tracker bias, which represents the altimeter height estimation error.

The radar system PTR is primarily the transmitted radar pulse shape. For an idealized, linear, frequency-modulated altimeter radar pulse, the radar system PTR is given by (Ulaby et al., 1982; Rodriguez, 1988):

\[ \chi(t) \sim \frac{\sin^2[(at/2)(T-|t|)]}{(at/2)^2} \quad -T \leq t \leq T \]

(2.4)

where \( 2T \) is the radar pulse length and \( a \) is a constant which depends on the radar bandwidth.

Figure 2.1 shows the PTR using TOPEX bandwidth \( B = 320 \text{ MHz} \), the surface impulse response using TOPEX orbit configurations, the specular point PDF using parameters \( z_r = 0, \sigma = 1 \text{ m}, \lambda = 0.4 \), and the convolved power which is computed using the convolution theorem.
2.3 Waveform Retracking Methods

2.3.1 Ocean Waveform Retracking – Deconvolution Method

Over ocean surfaces, altimeter retracking algorithms aim to retrieve more accurate estimates of the range and the SWH, as well as additional parameters such as the antenna off-nadir angle and the skewness of the surface elevation distribution. Ocean waveform retracking is based upon the convolution representation of the waveform. Therefore, one can recover the specular point PDF by performing a deconvolution of the return radar waveform (Barrick and Lipa, 1985; Rodriguez and Chapman, 1989). Due to the fact that the specular point PDF is a function of the SWH, the ocean surface skewness, and the
altimeter tracker error, one can estimate these parameters by performing a functional fit to the deconvolved PDF (Rodriguez and Chapman, 1989). Equation (2.1) can be expressed as a matrix equation \( y = M x \), where \( M \) is \( S(t) \otimes \chi(t) \) and \( x \) is \( f_{sp}(t) \). However, this equation is effectively singular, and Rodriguez and Chapman (1989) suggested a modified convolution model, which is called the "corrected slopes convolution model". It starts with a simpler \( S(t) \) where the Bessel function \( I_0(\beta t^{1/2}) \) is replaced by \( \exp(\beta^2 t / 4) \) such as:

\[
S(t) = A \exp\left[-\left(\alpha - \frac{\beta^2}{4}\right)t\right] U(t)
\] (2.5)

Then, the derivative of the return power can be expressed as:

\[
P'(t) = [\chi(t) \otimes f_{sp}(t)] \otimes S'(t)
\]

\[
= [\chi(t) \otimes f_{sp}(t)] \otimes \left[A \delta(t) - \left(\alpha - \frac{\beta^2}{4}\right) S(t)\right]
\]

\[
= A [\chi(t) \otimes f_{sp}(t)] - \left(\alpha - \frac{\beta^2}{4}\right) P(t)
\] (2.6)

\[
\Rightarrow \chi(t) \otimes f_{sp}(t) = \frac{1}{A} \left[P'(t) + \left(\alpha - \frac{\beta^2}{4}\right) P(t)\right]
\]

where \( \delta(t) \) is a Dirac delta function.

After approximation of the convolution integral using numerical quadrature rule, the specular point PDF can be solved by least squares algorithm.

\[
x = \left[M^T M\right]^{-1} M^T y
\] (2.7)

Due to the fact that this equation is unstable, i.e., sensitive to noise, Twomey’s regularization method (1962) for solving Equation (2.6), which is Fredholm integral equation of the first kind, is used. Once the deconvolved PDF is obtained, the unknown parameters \( z_r, \lambda, \sigma \) and \( A \) are solved by linearizing the specular PDF (Lipa and Barrick, 1981; Rodriguez 1988) or applying the more robust Levenberg-Marquardt method (Rodriguez and Chapman, 1989).

2.3.2 NASA V4 (\( \beta \)-) Retracker
The β-retracker is the first retracking algorithm developed to obtain corrected ranges from SEASAT-1 radar altimeter over the Antarctic and Greenland continental ice sheets (Martin et al., 1983). This algorithm uses 5- or 9-parameter functions to fit a single- or double-ramped waveform, respectively. The double-ramped waveforms can be found when two distinct surfaces at different elevations exist within the range window. The Ice Altimetry Group of NASA’s Goddard Space Flight Center (GSFC) has developed algorithms for retracking polar ice altimetry based on Martin’s functions. There have been four versions of the GSFC retracking algorithms (Zwally, 1996), and Version 4 employs an exponential function instead of a linear function to fit a fast-decaying trailing edge which is caused by the beam attenuation over sea ice or ice sheets:

\[
y(t) = \beta_1 + \beta_2 e^{-\beta_3 t} P\left(\frac{t - \beta_3}{\beta_4}\right)
\]

where

\[
Q = \begin{cases} 
0 & \text{if } t < \beta_3 - 2\beta_4 \\
\frac{1}{t - (\beta_3 - 2\beta_4)} & \text{if } t \geq \beta_3 - 2\beta_4
\end{cases}
\]

\[
P(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-q^2} dq \quad \text{with } q = \frac{t - \beta_3}{\beta_4}
\]

The unknown parameters are as follows:

- \(\beta_1\): the thermal noise (or DC level) of the waveform.
- \(\beta_2\): waveform amplitude.
- \(\beta_3\): mid-point of the leading edge which marks the correct time delay.
- \(\beta_4\): slope of the leading edge which is related to SWH.
- \(\beta_5\): slope of the trailing edge which is related to the scattering at the footprint.

The range correction from the retracking algorithm is obtained by the difference between the estimated mid-point of the leading edge and the on-board tracking gate (23.5 for TOPEX, 32.5 for ERS-1/2) multiplied by the distance which is related to a single gate such as:

\[
\Delta R = (\beta_3 - \text{tracking gate}) \times \Delta s
\]

where

\[
\Delta s = t \times c / 2 = 0.4684 \text{ m}
\]

where \(t\) is pulse width (3.125 nsec for TOPEX) and \(c\) is speed of light.
Figure 2.2: (a) 5-parameter retracker model, (b) An example of a land-returned waveform and the fitted 5-parameter single ramp function.

It should be noted that the on-board 128-sample TOPEX waveform, which has uniform gate interval of 3.125 nsec with its tracking point at sample (or gate) 32.5, is averaged in multiples of 1, 2, and 4 to form the 64-sample telemetry waveform which is summarized in Table 2.1. Waveform samples 17-48 in 128-sample waveform are not combined (averaged) to preserve the original sampling rate (3.125 nsec) across the leading edge of the waveform. Therefore, Equation (2.9) is valid when the retracked gate lies between 9 and 40.
<table>
<thead>
<tr>
<th>128-sample waveform</th>
<th>64-sample waveform</th>
<th>sampling rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-16</td>
<td>1-8</td>
<td>averaged by two</td>
</tr>
<tr>
<td>17-48</td>
<td>9-40</td>
<td>one to one match</td>
</tr>
<tr>
<td>49-64</td>
<td>41-48</td>
<td>averaged by two</td>
</tr>
<tr>
<td>65-128</td>
<td>49-64</td>
<td>averaged by four</td>
</tr>
</tbody>
</table>

Table 2.1 TOPEX telemetry sample to waveform sample relationship (Hayne et al., 1994).

As mentioned above, some of the waveforms obtained from ice sheets may include the second ramp due to a distinct height change within the footprint, which can be fitted by 9-parameter model:

$$y = \beta_1 + \beta_2 (1 + \beta_3 Q_1) e^{\left(\frac{t - \beta_3}{\beta_4}\right)} + \beta_5 e^{-\beta_6 Q_2} e^{\left(\frac{t - \beta_6}{\beta_7}\right)}$$  \hspace{1cm} (2.10)

where

$$Q_1 = \begin{cases} 
0 & \text{for } t < \beta_3 + .5\beta_4 \\
(t - (\beta_3 + .5\beta_4)) & \text{for } t \geq \beta_3 + .5\beta_4 
\end{cases}$$

$$Q_2 = \begin{cases} 
0 & \text{for } t < \beta_6 + .5\beta_7 \\
(t - (\beta_6 + .5\beta_7)) & \text{for } t \geq \beta_6 + .5\beta_7 
\end{cases}$$

$$P(z) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{q^2}{2}} dq$$

As a result, there are two corrections each representing an independent range from the satellite to the surface such as (Martin et al., 1983):

$$\Delta R_1 = (\beta_3 - \text{tracking gate}) \times \Delta s$$
$$\Delta R_2 = (\beta_6 - \text{tracking gate}) \times \Delta s$$  \hspace{1cm} (2.11)

where $\beta_3$ and $\beta_6$ are the locations of the center of the first and second ramps, respectively.

The retracked gates are then

$$R_1 = R_{\text{observed}} + \Delta R_1$$
$$R_2 = R_{\text{observed}} + \Delta R_2$$  \hspace{1cm} (2.12)
Figure 2.3: (a) 9-parameter retracker model, (b) example of a double-ramp waveform and the fitted 9-parameter waveform model.

2.3.3 Offset Center of Gravity (OCOG) Retracker

The OCOG algorithm was developed as an empirical technique to produce ice sheet data products from ERS-1/2 radar altimetry. It estimates an amplitude of the waveform and uses this to threshold retrack the leading edge (Bamber, 1994). It calculates the center of gravity of a rectangular box and the amplitude is twice the height of the center of gravity. The squares of the sample values are used to reduce the effect of low amplitude samples in front of the leading edge.
Figure 2.4: Schematic description of the OCOG retracker.

\[
COG = \frac{\sum_{i=1+n_a}^{64-n_a} iP^2(i)}{\sum_{i=1+n_a}^{64-n_a} P^2(i)}
\]

\[
Amplitude = \sqrt{\frac{\sum_{i=1+n_a}^{64-n_a} P^4(i)}{\sum_{i=1+n_a}^{64-n_a} P^2(i)}}
\]

\[
Width = \left(\frac{\sum_{i=1+n_a}^{64-n_a} P^2(i)}{\sum_{i=1+n_a}^{64-n_a} P^2(i)}\right)^2
\]

where \( P(i) \) is the waveform sample value at the \( i^{th} \) bin, and \( n_a \) is the number of aliased sample (\( n_a = 4 \) for TOPEX). In addition, the waveform samples 45-50 are excluded to avoid the leakage effects of TOPEX waveforms (Hayne et al., 1994).

Finally, the leading edge position (\( LEP \)) is given by,

\[
LEP = COG - Width/2
\]

2.3.4 Threshold Retracker

The threshold retracking algorithm was developed primarily to measure ice sheet elevation change (Davis, 1997). The leading edge position is determined by locating the
first waveform sample to exceed the percentage (i.e., threshold level) of the maximum waveform sample amplitude. The pre-leading edge DC level (or thermal noise) is computed by averaging the waveform sample 5 to 7, and again, the samples from 1 to 4, 45 to 50, and 61 to 64 are excluded for TOPEX Ku-band waveform. Davis (1997) suggests the 50% threshold for surface-scattering dominated waveforms, and 10% or 20% threshold level for volume-scattering surface.

\[ A_{\text{max}} = \max(P(i)) \]
\[ DC = \frac{1}{3} \sum_{i=5}^{7} P(i) \]
\[ TL = DC + T_{\text{coeff}} (A_{\text{max}} - DC) \]

with:
- \( A_{\text{max}} \): maximum waveform amplitude;
- \( DC \): thermal noise or DC level;
- \( T_{\text{coeff}} \): threshold level;
- \( TL \): retracked gate.

2.4 Surface Gradient Error

One issue that must be considered is the effect of the surface gradient in generating a time series of the surface height differences from the repeat-orbit altimeter data as the horizontal location of each observation point changes from cycle to cycle. If we select relatively flat land surfaces (standard deviation of the height variation < 40 cm) around land regions near Hudson Bay as in Lee et al. (2008a), we can model the surface as a plane and the surface gradient can be computed from the satellite observations, i.e., the retracked surface heights. The detailed gradient estimation algorithm can be found in Guman (1997). It should be emphasized that this approach is valid only over reasonably flat surfaces. The local mean land surface in a study region (say, bin) can be modeled with along- and cross-track gradients as:

\[ LSH = a + b \cdot dx + c \cdot dy \]

with:
- \( LSH \): mean land surface height;
\( a \): height of the plane at the bin center;

\( b \): along-track land surface gradient;

\( c \): cross-track land surface gradient;

\( dx \): along-track displacement of a data point from the bin center;

\( dy \): cross-track displacement of a data point from the bin center.

The parameters \( a, b \) and \( c \) can be estimated by the least squares adjustment. However, in addition to the spatial variation, the land surface height may also exhibit temporal changes from cycle to cycle. Thus, a model with six parameters that includes linear, annual, and semi-annual variations is adopted:

\[
LSH = A + B \cdot (t - t_0) + c_1 \cdot \cos[\omega(t - t_0)] + s_1 \cdot \sin[\omega(t - t_0)] +
\]

\[
c_2 \cdot \cos[2\omega(t - t_0)] + s_2 \cdot \sin[2\omega(t - t_0)]
\]  \hspace{1cm} (2.17)

with:

\( A \): offset or bias;

\( B \): linear slope;

\( c_1, c_2 \): amplitudes of the cosine term;

\( s_1, s_2 \): amplitudes of the sine term;

\( \omega \): annual frequency.

To reduce the effect of the terrain surface gradient on the estimate of the annual/semi-annual variations, Equations (2.16) and (2.17) are implemented in an iterative scheme. First, a preliminary annual/semi-annual variability is estimated using Equation (2.17), and then the preliminary land surface gradient using the annual/semi-annual variation-removed surface height is estimated. This procedure is iterated to yield a gradient-corrected surface height. However, this approach would be no longer available over rougher surfaces, and the surface gradient correction employed in this study using a high-resolution DEM will be presented in Chapter 3.5.