Variations of Continental Ice Sheets Combining Satellite Gravimetry and Altimetry

by

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Variations of Continental Ice Sheets Combining Satellite Gravimetry and Altimetry

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By

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Abstract

Knowledge of mass variations of continental ice sheets including both Greenland ice sheet (GrIS) and the Antarctic ice sheet (AIS) is important for quantifying their contribution to present-day sea level rise and predicting their potential changes in the future. Previous estimates for the respective trend of mass variations over both ice sheets using the Gravity Recovery and Climate Experiment (GRACE) data differ widely, primarily contaminated by the large uncertainties of glacial isostatic adjustment (GIA) models over Antarctica, inter-annual or longer variations due to the relatively short data span, and also limited by the coarse spatial resolution (~333 km) of the GRACE data. Satellite radar altimetry such as the Environmental satellite (Envisat) altimetry measures ice sheet elevation change with much better spatial resolution (~25 km) especially over polar region but has no information about the density of ice/snow associated with measured elevation change to infer mass change. In this study, the mass variations derived from GRACE data and elevation change from Envisat data are investigated by decomposing them separately into three components, the trend, seasonal and inter-annual variations, with the focus on combining the variations of mass change from GRACE and elevation change from Envisat at the inter-annual scale to estimate the nominal density of ice/snow associated with measured inter-annual mass and elevation changes.
To process Envisat data, the along-track repeat analysis with surface gradient corrected by the use of digital elevation model (DEM) and the collinear method are modified separately. By comparing the root mean square (RMS) of elevation changes generated from both methods, the trend maps of elevation changes, and their corresponding uncertainties, the modified collinear method is demonstrated to be more effective than the use of DEM for surface gradient correction. Moreover, the elevation change retrieved by the ice-1 and ice-2 waveform retracking algorithms are compared in three cases over the AIS, with results confirming that for the ice-2 algorithm, only backscatter correction is not enough, the correction from changes in waveform shape parameters is needed. The trend of elevation change retrieved by the ice-1 algorithm with only backscatter analysis is consistent with that of elevation change retrieved by the ice-2 algorithm with correction from changes in backscattered power and waveform shape parameters. The trends from the monthly output of the Regional Atmospheric Climate Model (RACMO2), GRACE data, and Envisat data, are respectively compared over both ice sheets. For the seasonal component, the amplitude and phase are depicted over the GrIS using these three datasets; seasonal variations from GRACE without and with the replacement of $c_{20}$ are compared over the AIS. At the inter-annual scale, mass change from GRACE data, elevation change from Envisat data, and surface mass change from the RACMO2 models are respectively extracted and compared over both ice sheets. For the GrIS, highly positive correlations are found mostly over the North, West, and parts of the East region; weakly positive correlations occur in Northwest, Southeast, and parts of Northeast; negative correlations appear in a small part of Northeast. For the AIS, highly positive
correlations are found mostly over the West and parts of the East region; weakly positive and negative correlations occur primarily in the East part. Furthermore, based on the assumption that both techniques can sense the same geophysical process, by combining the two inter-annual variations, nominal densities are estimated separately over 9 regions in the GrIS and 14 regions in the AIS. Over the GrIS, the nominal densities range from $299.15\pm11.2$ kg/m$^3$ to $764.57\pm43.4$ kg/m$^3$ over 7 regions, and are physically unrealistic over the other 2 selected regions. Based on the Envisat data corrected by the estimated density value, the inter-annual mass variations with high spatial resolution are obtained over one example region in Southwest GrIS. For the AIS, the estimated nominal densities range from $317.49\pm24.2$ kg m$^3$ to $479.81\pm25.3$ kg m$^3$ over 4 regions with all correlation coefficients larger than 0.7; the nominal density is $450.17\pm28.8$ kg m$^3$ during 2003–2006 and $812.45\pm44.3$ kg m$^3$ during 2007–2009, and exhibits time dependence over the Amundsen Sea (AS) sector; for another 4 selected regions, the estimated densities are slightly smaller than snow density. For the other regions, the estimated densities are physically unrealistic. We find that the area of negative correlations representing the inconsistency between the two independent observation types, GRACE and Envisat, is now only 21% over Antarctica, showing a significant reduction of 40%, compared with the results from a previous study.
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Chapter 1: Introduction

1.1 Sea Level Rise

Sea level is a sensitive indicator for potential climate change. It primarily fluctuates with geophysical contributions from several processes (Shum and Kuo, 2011), including changes on ocean temperature and salinity (Antonov et al., 2005; Ishii et al., 2006; Roemmich and Owens, 2000), melting of mountain glaciers and ice sheets (Chen et al., 2006, 2009; Meier et al., 2007; Velicogna, 2009), and the increased or decreased runoff from the land hydrologic cycle (Cazenave et al., 2000), as well as the deformation of solid Earth caused by glacial isostatic adjustment (GIA) due to the deglaciation of the last ice age (Douglas and Peltier, 2002; Tamisiea, 2011) and also by the ongoing deepening of ocean basins (Tamisiea and Mitrovica, 2011).

Using historical tide gauge data, Church and White (2006) estimated a rate of sea-level rise of $1.7 \pm 0.3$ mm/yr over the 20th century. They also claimed the detection of an acceleration of $0.013 \pm 0.006$ mm/yr$^2$ for sea level rise. According to the 4th Assessment Report (AR4) of the Intergovernmental Panel on Climate Change (IPCC) (Bindoff et al., 2007), during the period 1993-2003, the altimetry-based rate of sea level rise is $3.1 \pm 0.4$ mm/yr, which can be partly explained by steric sea level rise at $1.6 \pm 0.25$ mm/yr and by land ice contributions of $1.2 \pm 0.2$ mm/yr. That is, during this 10-year period, half of the rate of sea level rise could be attributed to steric sea level rise primarily caused by ocean thermal expansion, with the remaining rate of rise from the shrinking of land ice. Lemke et al. (2007) mentioned in IPCC AR4 that for 1993-2003, the estimated contributions from the Greenland ice sheet (GrIS) to sea level was $0.21 \pm 0.035$ mm/yr, and $0.21 \pm 0.17$ mm/yr from the Antarctic ice sheet (AIS). Using satellite altimetric data during years 2003-2008, Cazenave et al. (2009) reported a reduced rate of sea level rise of $2.5 \pm 0.4$ mm/yr from altimetry. They also estimated the contribution of $2.3 \pm 0.3$ mm/yr from ice sheets, Glaciers and ice caps, and terrestrial waters to sea level rise, using the Gravity Recovery and Climate Experiment (GRACE) data (Tapley et al., 2004) during the same period; and they estimated a rate of $0.37 \pm 0.1$ mm/yr steric sea level rise from Argo data during 2004-2008. They concluded that the reduced rate of sea level rise might be due to the break of ocean heat content since 2003 (Willis et al., 2008), and they showed that about half of the increase in ocean mass since 2003 was because of an enhanced contribution from polar ice sheets, with the other half from the melting of
mountain glaciers and ice caps. *Chen et al. (2013)* estimated a rate of $0.60 \pm 0.27 \text{ mm/yr}$ for steric sea level rise from the Argo data, and a rate of $1.80 \pm 0.47 \text{ mm/yr}$ from GRACE gravity data with contributions of $-0.69 \pm 0.05 \text{ mm/yr}$ from the GrIS and of $-0.50 \pm 0.26 \text{ mm/yr}$ from the AIS, yielding a rate of $2.40 \pm 0.54 \text{ mm/yr}$ for the global mean sea level rise during years 2005-2011. Their results agreed well with the trend of $2.39 \pm 0.48 \text{ mm/yr}$ obtained from altimetry during the same period.

Comparing the contribution from ice sheets to sea level rise during years 2005-2011 reported by *Chen et al. (2013)* with those given by *Lemke et al. (2007)* and *Cazenave et al. (2009)* in previous years, we have to agree that for years 1993-2011, the GrIS and AIS have an accelerating contribution to sea level rise. If the result of steric sea level rise reported by *Chen et al. (2013)* was accurate, the influence from the melting ice sheets and mountain glaciers are three times larger than that from the steric sea level contributions on the global sea level rise during 2005-2011. It should be noted that since the 7-year period is relatively short, the estimates of sea level rise rates from *Chen et al. (2013)* could be contaminated by the strong inter-annual or longer variability.

### 1.2 Satellite Gravimetry and Altimetry

Knowledge of the Earth’s static gravity field and its temporal component are important to understand the configuration of masses (the solid Earth, ocean, cryosphere, hydrosphere, and atmosphere) and their interactions with some geophysical processes, i.e., hydrologic cycle, cryosphere, ocean circulation, and tectonics (*Wahr et al., 1998*).

With the successful operations of the dedicated satellite gravity missions, CHAllenging Minisatellite Payload (CHAMP) (*Reigber et al. 2003*), GRACE, and Gravity and Ocean Circulation Explorer (GOCE) (*ESA, 1999*), considerable improvements have been achieved on the observations and modeling of the Earth’s gravity field and its time variable components. Particularly, with the GRACE mission, time-varying gravity field has become an important subject of geodetic research (*Flury and Rummel, 2005*), capable of research on diverse disciplines including climate change, hydrology, glaciology, geophysics, and oceanography, as well as other branches of Earth sciences (*Wahr et al., 1998*).

GRACE twin satellites were launched on March 2002 and jointly operated by the U.S. National Aeronautics and Space Administration (NASA) and the German Aerospace Center DLR with a designed lifetime of 5 years (*Tapley et al., 2004*). Both satellites are flying in the same polar orbit with an altitude of about 500 km and a satellite-to-satellite distance of about 220 km. GRACE uses a state-of-the-art technique by measuring the inter-satellite range and range rate between the twin satellites to detect the variation of the Earth’s gravity field with unprecedented accuracy. Until now, GRACE has been running for more than 10 years, with fruitful achievements from its wide applications to multi-disciplines (*Tapley et al., 2014*).
As primarily limited by the altitude of satellite orbit and the distance between the twin satellites, the spatial resolution of GRACE is relatively coarse, i.e., a few hundred kilometers (> 333 km). Because of the inter-satellite tracking measurements are limited to the along-track direction, resulting in rank deficiency on the ensuring gravity field inversion causing spatially correlated high-frequency errors in the form of north-south stripes. Thus, post-processing including decorrelation, spatial smoothing or filtering and signal leakage reduction are needed to remove the north-south stripes (Swenson and Wahr, 2006; Duan et al., 2009; Guo et al., 2010). Over ice sheets, the mass variations derived from the GRACE data suffer from the uncertainties of GIA models (Guo et al., 2012), primarily for the AIS. The GIA effect over Antarctica is of the same order of magnitude as present-day ice mass variations (Ivins and James, 2005; Peltier, 2009).

Satellite altimetry missions have been launched since the mid-1970s with the initial primary objectives to observe the ocean surface topography. The onboard satellite radar altimeter is generally used to measure the range from the satellite to the ocean surface on the nadir direction. With the determined orbit altitude and the corrections, i.e., the medium correction and the geophysical corrections, the ocean surface height can be obtained relative to a reference ellipsoid. The early spaceborne altimeters were limited primarily by the uncertainties in the orbit determination, instrumental errors, atmospheric refraction, and the interaction between the radar pulse and the air-sea interface, as no continuous direct observations about the satellite position were available and only corrections based on relatively crude models and inaccurate observations could be applied (Fu et al., 1988). Over the first 15 years history of satellite altimetry, as the knowledge of the Earth’s gravity field was being improved, the accuracy of satellite orbit determination was improved from 10 m to 50 centimeters or even better (Marsh et al., 1988).

Since the early 1990, satellite altimetry has become an important tool to continuously measure the ocean surface, ice sheets and hydrologic bodies (Shum et al., 1995). For oceanic applications, data from the TOPEX/Poseidon mission launched in 1992 are mainly preferred, because of the dual-frequency radar altimeter instrument, and its optimal design to minimize orbit errors and reducing tidal aliasing. In order to continuously monitor the global sea level variation, Jason-1 and Jason-2 satellites followed the TOPEX/Poseidon satellite in 2001 and 2008 separately, with the same orbit inclination of 66°, 1330 km orbital altitude and 10-day repeat orbits (Cazenave et al., 2009). Different from the satellite series of the TOPEX/Poseidon, the European Remote Sensing 1 (ERS-1) was launched in 1991 with an orbital inclination of 98.5°, which provided measurements by specifically operating in coarse-track mode (ice mode), as opposed to the ocean mode, over most of polar regions including the whole GrIS and the AIS with latitude higher than 81.5°S (Fu and Cazenave, 2001). The follow-on satellites for the ERS-1 include ERS-2 and the Environmental Satellite (Envisat) launched in 1995 and 2002 respectively.
As mentioned above, satellite radar altimetry was initially designed to investigate variations on the ocean surface including ocean circulation, ocean tides, and global sea level change and so on, and it has shown potential applications over non-ocean surfaces. For regions with rugged terrain, the onboard altimeter might lose tracking, as the onboard tracker fails in precisely predicting the corresponding range. For a relatively flat surface, the altimeter is able to maintain tracking but the measured range can be difficult to be accurately retrieved because the echo returned from the Earth’s surface includes the part reflected away from the nadir point (Fu and Cazenave, 2001). In such cases, the range measurements need to be corrected by ground-based processing (retracking). Several retracking algorithms have been developed by researchers to investigate the elevation change over continental ice sheets; Martin et al. (1983) developed algorithms with 5 and 9 parameters corresponding to different shapes of echoes from Seasat-1 radar altimeter; Wingham et al. (1986) performed robust retracking by adopting the Offset Center of Gravity (OCOG) algorithm; Davis (1995, 1996, 1997) adopted and compared the threshold retracking with different threshold levels; Legresy and Remy (1997) investigated the ice-2 retracking algorithm which is optimized for ocean-like echoes. Based on these retracking algorithms, the elevation change of continental ice sheets was retrieved and investigated (Zwally et al., 1989; Wingham et al., 1998; Davis and Ferguson, 2005).

Besides the above conventional radar altimeter, the newly developed delay/Doppler altimeter instrument onboard Cryosphere Satellite 2 (CryoSat-2) launched in 2010 is dedicated to measure continental ice sheets and sea ice freeboard change (Raney, 1998; Wingham et al., 2006b). For satellite altimetry with 35-day repeat orbits, Satellite with ARgos and ALtika (SARAL) was launched in 2013 with the Altika altimeter working in Ka-band (35GHz).

In addition, satellite laser altimetry, the Geoscience Laser Altimeter System launched in 2002 on board Ice, Cloud and land Elevation Satellite (ICESat), was also developed to primarily investigate the elevation changes of continental ice sheets (Zwally et al., 2002; Shuman et al., 2006).

1.3 Motivation for this Study

The continental ice sheets (the GrIS and the AIS), are of great concern in research related to global climate change and present-day sea level rise, as well as regional climate modeling (Helsen et al., 2008; van den Broeke et al., 2011; Hanna et al., 2013). Ice sheets are sensitive to and respond continuously to climate change. If both ice sheets melt completely, the global mean sea level would rise up by more than 70 m (Zwally et al., 2002). The GrIS experiences widespread summer melting depending on the temperature in the summer. It suffers from outlet glaciers of fast speed, with large volume of ice draining into the ocean. In contrast, the AIS experiences little surface melting. It loses ice primarily by basal melting and iceberg calving at the ice shelves.
To assess mass variations of both ice sheets and their contribution to sea level change, studies have been conducted primarily using one or two of the following three different approaches: the mass budget method (van den Broeke et al., 2009); satellite gravimetry (Chen et al., 2006); and satellite altimetry (Zwally et al., 1989). The mass budget method combines estimates of surface mass balance (SMB) and ice discharge. Hence, the technique is vulnerable to inaccuracies in either of the two glaciological variables (van den Broeke et al., 2011), although it can be used to partition the total mass variations of ice sheets (van den Broeke et al., 2009; van Angelen et al., 2014). GRACE mission provides direct observations of the Earth’s temporal gravity field, which enable the inference of surface mass change. However, its current data span of over 10 years is not long enough to accurately estimate the rate of ice sheet mass variations (Wouters et al., 2013). Published trend estimates of GRACE-observed ice sheet mass variations differ widely and have large uncertainties, and could be significantly contaminated by inter-annual (Sasgen et al., 2012; van den Broeke et al., 2011) variations since the time period of data applied in these studies are relatively short. The main limitations of GRACE data include the large uncertainties currently existing in GIA models over Antarctica, and coarse spatial resolution (>333 km). Satellite altimetry measures the surface elevation change at much higher spatial resolutions (~2-30 km in polar region), and is capable of readily inferring volume change, except for regions with relatively large terrain gradients. With proper knowledge of densities corresponding to the measured volume changes, the mass variations can be estimated using altimetric measurements. However, ice/firn density at which the measured elevation takes place for both ice sheets is not well known and widely different density values are assumed by prior studies. Consequently, these different density assumptions cause discrepancy between the computed mass variations, even if the same volume changes are derived from satellite altimetry. This situation might get worse if mass variations occur with smaller magnitude but over a large area.

Besides, for the processing of altimetric data, the crossover method (Wingham et al., 1998; Davis et al., 2005; Yang et al., 2013) and the along-track repeat method (Legresy et al., 2006; Lee et al., 2012; Flament and Remy, 2012) are commonly used. The former can minimize the topographic induced errors but it limits the number of valid observations. Other factors, i.e., antenna polarization, also degrade the accuracy of crossover method. Legresy et al. (2006) showed that the along-track repeat method can provide about 100 times more individual measurements over the AIS, and it can enhance the spatial resolution when studying the variations in local region. In contrast to crossover method, it could mitigate the tracker biases (different radar altimeter echoes induced by ascending and descending passes, due to the non-symmetric altimeter antenna on ERS-1/2/Envisat). For the along-track repeat method, there are different approaches to remove the topographic effect. Legresy et al. (2006) utilized geographic function to model the surface gradient; modified collinear methods were used by Ewert et al. (2012), and Flament and Remy (2012); Slobe et al. (2008), Lee et al. (2012), and Su et al. (2015) used an external DEM to correct the surface gradient. In addition, different retracking algorithms, the ice-2 algorithm and the ice-1 algorithm, were used in previous studies (Legresy et al., 2006; Flament and Remy, 2012; Lee et al., 2012; Su et al., 2015). For the
ice-2 algorithm, the shape parameters of the echo are used together with backscattered power to remove the spurious elevation change linearly associated with changes on the shape parameters and backscattered power. For the ice-1 algorithm, only backscatter power analysis is used. According to Remy et al. (2012), there was a difference of 5 cm/yr on the trend of elevation change retrieved by only backscatter analysis and the analysis with backscatter power and shape parameters from the ice-2 algorithm over some regions in the AIS.

In this dissertation, the along-track analysis approach with surface gradient corrected by both use of DEM and collinear methods are modified separately and compared for Envisat altimetry data processing over both ice sheets. Applying the modified collinear method, for the ice-1 and ice-2 algorithms, elevation changes generated in three cases are analyzed over the AIS step by step: (1) only backscatter analysis for the ice-1 algorithm; (2) only backscatter analysis for the ice-2 algorithm; (3) backscatter analysis together with shape parameters for the ice-2 algorithm. Furthermore, the joint analysis are conducted by combining GRACE and Envisat data over both ice sheets separately, with the objective of estimating nominal density at the inter-annual scale purely using geodetic observations for the first time. In addition, considering that the variations of surface mass and elevation are associated with SMB variations, and that the geodetic observations have the potential to improve the SMB models, and better characterize spatial variations of SMB variations over both ice sheets.

1.4 Organization of this Dissertation

Chapter 2 reviews the formulations for deriving surface mass variation from GRACE Level 2 data, and introduces the post-processing techniques for GRACE Level 2 GSM data products.

Chapter 3 introduces the basic principle for satellite radar altimetry, the algorithms used to retrieve height change, and the along-track repeat method for processing altimetric data over ice sheets.

Chapter 4 shows the variations of SMB from the Regional Atmospheric Climate Model, RACMO2/GR, in two different versions over the whole Greenland and 7 basins separately. The features of SMB and the total mass variations derived from GRACE data are investigated in three components, the trend, seasonal and inter-annual components, respectively. The cumulative mass changes obtained from the mass budget method and GRACE data are compared over the GrIS. Using Envisat altimetric data, the elevation change is studied in three components in the GrIS. Finally, nominal densities are estimated over the selected regions in the GrIS, combining inter-annual variations of mass change from GRACE and elevation change from Envisat. Based on the altimetric data with density corrected, inter-annual mass variations with higher resolution are computed and shown over an example region over Greenland.
Chapter 5 depicts the characteristics of SMB from the Regional Atmospheric Climate Model, RACMO2/ANT, in two different versions, and the total mass variations obtained from GRACE data in three components over the Antarctica. Using Envisat data, the along-track repeat methods with surface gradient correction by the use of DEMs and modified collinear approach are compared; the height retrieved from the ice-1 and ice-2 algorithms with three ways of analysis are further compared. Finally, a joint analysis of GRACE and Envisat data is conducted to estimate the nominal density over selected regions in the AIS.

Chapter 6 summarizes the conclusions in this dissertation, and the further investigation in the future is suggested.
Chapter 2: GRACE Data Processing

2.1 Surface Mass Variation Derived from Temporal Gravity Change

Referring to Wahr et al. (1998), the method used to infer surface mass variations from gravity information provided by GRACE data can be derived like the following.

The Earth’s global gravity field can be commonly described by the shape of the geoid, which is an equipotential surface closely approximating the mean sealevel. The geoid undulation $N$ can be expressed in terms of spherical harmonics:

$$N(\theta, \lambda) = a \sum_{l=0}^{\infty} \sum_{m=0}^{l} (\tilde{C}_{lm} \cos m\lambda + \tilde{S}_{lm} \sin m\lambda) \bar{P}_{lm}(\cos \theta)$$

(2.1)

where $a$ is Earth’s equatorial radius, $\theta, \lambda$ are the colatitude and longitude, $l$ and $m$ are the degree and order of the harmonic coefficients, respectively $0 \leq m \leq l, l \geq 0$. $\tilde{C}_{lm}, \tilde{S}_{lm}$ are the normalized spherical harmonic coefficients, $\bar{P}_{lm}(\cos \theta)$ is the fully-normalized associated Legendre functions (of the First Kind) with degree $l$ and order $m$.

Assume $t = \cos \theta$, then:

$$\bar{P}_{lm}(t) = \begin{cases} \sqrt{2(l+1)\frac{(l-m)!}{(l+m)!}} P_{lm}(t), & 0 < m \leq l \\ \sqrt{2(l+1)} P_{l0}(t), & m = 0 \end{cases}$$

(2.2)

$$P_{lm}(t) = \frac{1}{2l!} (1 - t^2)^{m/2} \frac{d^{l+m}}{dt^{l+m}} (t^2 - 1)^l$$

(2.3)

For GRACE mission, the monthly gravity field solutions consist of spherical harmonics with degree and order complete to 60.

Assume that there is a time-dependent change of the geoid $\Delta N$ due to the mass redistribution within the Earth. Such a change is related to the corresponding density redistribution $\Delta \rho (r, \theta, \lambda)$, and can be obtained by changes, $\Delta \tilde{C}_{lm}$ and $\Delta \tilde{S}_{lm}$, in the corresponding spherical harmonic coefficients. According to Chao and Gross (1987), we have:
where \( \alpha \) is the mean equatorial radius of the Earth, \( \rho_a \) is the average density of Earth (i.e., 5517 kg\cdot m^{-3}), \( \Delta \rho(r, \theta, \lambda) \) is the density redistribution causing such a geoid change, and \( r \) is the distance between the calculated point and the center of the Earth.

Suppose the portions of significant mass variations within the Earth including atmosphere can be approximated as a thin layer, \( \Delta \rho \) is then concentrated in such a thin layer of thickness \( \tau \) at the Earth’s surface. For recovering the surface mass redistribution from the measured temporal gravity signals, this layer must be thick enough to include the atmosphere, oceans, ice caps, and terrestrial water storage with significant mass fluctuations.

Thus, the change in surface density (i.e., mass/area) can be defined as the radial integral of \( \Delta \rho \) through this thin layer:

\[
\Delta \sigma(\theta, \phi) = \int_{\text{thin layer}} \Delta \rho(r, \theta, \phi) dr
\]  

(2.5)

Considering the accuracy of measurements from GRACE, the monthly gravity field solutions in terms of spherical harmonic coefficients are truncated to degree \( l = 60 < l_{max} \), including most of sensed temporal gravity signals originating from the recoverable signals of mass redistribution. Suppose \( H \) is thin enough that \( (l_{max} + 2)H/a \ll 1 \). Then, \( (r/a)^{l+2} \approx 1 \), and Eq. (2.4) can be rewritten:

\[
\begin{bmatrix}
\{ \Delta C_{lm} \} \\
\{ \Delta S_{lm} \}_{\text{surface mass}}
\end{bmatrix} = \frac{3}{4\pi \alpha \rho_a (2l+1)} \int \Delta \sigma(\theta, \phi) \begin{bmatrix}
\cos(m\lambda) \\
\sin(m\lambda)
\end{bmatrix} \sin \theta d\theta d\phi d\phi
\]

(2.6)

This equation represents the contribution to the geoid from the direct gravitational attraction of the surface mass redistribution. In addition, the redistributed surface mass also loads and deforms the underlying solid Earth and thus causes an additional geoid contribution:

\[
\begin{bmatrix}
\{ \Delta C_{lm} \} \\
\{ \Delta S_{lm} \}_{\text{solid Earth}}
\end{bmatrix} = \frac{3}{4\pi \alpha \rho_a (2l+1)} \int \Delta \sigma(\theta, \phi) \begin{bmatrix}
\cos(m\lambda) \\
\sin(m\lambda)
\end{bmatrix} \sin \theta d\theta d\phi d\phi
\]

(2.7)

where \( k_l \) is the load deformation Love number of degree \( l \) (Munk and Macdonald, 1960). The total geoid change is the sum of these two components and can be expressed like this:

\[
\begin{bmatrix}
\{ \Delta C_{lm} \} \\
\{ \Delta S_{lm} \}
\end{bmatrix} = \begin{bmatrix}
\{ \Delta C_{lm} \} \\
\{ \Delta S_{lm} \}_{\text{surface mass}}
\end{bmatrix} + \begin{bmatrix}
\{ \Delta C_{lm} \} \\
\{ \Delta S_{lm} \}_{\text{solid Earth}}
\end{bmatrix}
\]
To summarize the above formulations for $\tilde{\sigma}$ and $\tilde{\tau}$ in a more compact form, $\Delta \sigma$ is expanded in terms of spherical harmonics $\tilde{C}_{lm}$ and $\tilde{S}_{lm}$:

$$\Delta \sigma(\theta, \lambda) = a \rho_w \sum_{l=0}^{\infty} \sum_{m=0}^{l} \tilde{P}_{lm}(\cos \theta) (\Delta C_{lm} \cos(m\lambda) + \Delta S_{lm} \sin(m\lambda))$$  \hspace{1cm} (2.9)

in which, $\rho_w$ is the density of water, $\Delta C_{lm}$ and $\Delta S_{lm}$ are dimensionless.

The normalized $\tilde{P}_{lm}$ satisfies the following condition:

$$\int_0^\pi \tilde{P}_{lm}^2(\cos \theta) \sin \theta \, d\theta = 2(2 - \delta_{m,0})$$  \hspace{1cm} (2.10)

from Eq. (2.9), we have:

$$\left\{ \frac{\Delta C_{lm}}{\Delta S_{lm}} \right\} = \frac{1}{4\pi \rho_w} \int_0^\pi \int_0^\pi \Delta \sigma(\theta, \lambda) \tilde{P}_{lm}(\cos \theta) \left\{ \cos(m\lambda) \right\} \sin \theta \, d\theta \, d\lambda$$  \hspace{1cm} (2.11)

Then, a simple relation between $\Delta C_{lm}$, $\Delta S_{lm}$, $\Delta \tilde{C}_{lm}$, $\Delta \tilde{S}_{lm}$ can be found:

$$\left\{ \frac{\Delta C_{lm}}{\Delta S_{lm}} \right\} = \frac{\rho_w}{3\rho_w} \frac{2l+1}{1+k_l} \left[ \frac{\Delta \tilde{C}_{lm}}{\Delta \tilde{S}_{lm}} \right]$$  \hspace{1cm} (2.12)

Thus, the change in surface density can be written by:

$$\Delta \sigma(\theta, \lambda) = a \rho_w \sum_{l=0}^{\infty} \sum_{m=0}^{l} \frac{2l+1}{1+k_l} \tilde{P}_{lm}(\cos \theta) (\Delta \tilde{C}_{lm} \cos(m\lambda) + \Delta \tilde{S}_{lm} \sin(m\lambda))$$  \hspace{1cm} (2.13)

More commonly, the change in surface mass $\Delta \sigma/\rho_w$ expressed in equivalent water thickness is used:

$$\Delta h(\theta, \lambda) = a \rho_a \sum_{l=0}^{\infty} \sum_{m=0}^{l} \frac{2l+1}{1+k_l} \tilde{P}_{lm}(\cos \theta) (\Delta \tilde{C}_{lm} \cos(m\lambda) + \Delta \tilde{S}_{lm} \sin(m\lambda))$$  \hspace{1cm} (2.14)

2.2 Post-processing Techniques

Using CSR release 05 (RL05) monthly gravity field solutions during years 2003-2009, and selecting the mean field (2003-2009) as the reference and following Eq. (2.14), the time series of surface mass variations can be computed. Arbitrarily, the mass variation in January 2006 is shown globally in Figure 2.1. From the figure, we can find that surface mass variation cannot be well observed because of the north-south stripes, which was reported in earlier studies using previous GRACE gravity field solutions. Similar cases can be found in all the other months. This indicates that large systematic errors are
present in the GRACE monthly gravity field solutions, especially in the resonant orders, i.e., 14, 28, and 42.

![Surface mass variation](image1)

Figure 2.1 Surface mass variation (referring to the mean field over 2003-2009) derived from GRACE RL05 monthly gravity field solutions without smoothing in January 2006.

There are formal standard deviations corresponding to spherical harmonic coefficients in the monthly GRACE gravity field solutions. Here, a plot of the standard deviations (Duan et al., 2009) indicating errors for all the spherical harmonic coefficients in the solution in January 2006 is depicted in Figure 2.2. Error pattern is evident, as shown by color coded values in Figure 2.2, i.e., smaller errors in spherical harmonic coefficients with degree ranging 10 to 50 and corresponding order ranging 0 to 15, and larger errors in sectorial harmonics and parts of tesseral harmonics. For zonal harmonics, relatively smaller error can be seen except for coefficients with degree less than 5 and larger than 45.

![Standard deviations](image2)

Figure 2.2 Formal standard deviations ($\times10^{-12}$) of estimated spherical harmonic coefficients in the monthly CSR RL05 GRACE gravity field solution for January 2006.
The degree variance can be defined by:

\[ D(l) = \sum_{m=0}^{l} (\Delta \tilde{C}_{lm}^2 + \Delta \tilde{\gamma}_{lm}^2) \]  \hspace{1cm} (2.15)

Thus, the degree variance of the signals can be computed using Eq. (2.15). Figure 2.3 shows the degree variance for the monthly GRACE gravity solution for January 2006. It can be seen that the degree variance decreases as the degree increases until the degree close to 30; after that, the degree variance increases gradually. The increasing degree variance in the high degrees is largely associated with the errors in these spherical harmonic coefficients in the GRACE solutions (Swenson and Wahr, 2011).

Figure 2.3 Degree variance (relative to the mean field during 2003-2009) of the monthly CSR RL05 GRACE gravity field solution for January 2006.

In order to reduce the errors in the GRACE gravity field solutions and improve the signal-to-noise ratio, various techniques have been developed for GRACE data post-processing by researchers (Wahr et al., 1998; Swenson and Wahr, 2006; Duan et al., 2009; Guo et al., 2010). In the following, 4 techniques, including replacing the long wavelength degree 2 spherical harmonic coefficients using an alternate solution, Gaussian smoothing, de-correlation, and signal leakage reduction, are described below.

2.2.1 Replacement of Spherical Harmonic Coefficient with Degree 2

For the GRACE RL05 gravity field solutions from CSR and JPL, the spherical harmonics with degree 2 and order 0, \( C_{20} \) should be replaced with the solution from Satellite Laser Ranging (SLR), as recommended by the GRACE data product Technical Note 07 (TN07). For GFZ GRACE RL05 data products, there is no replacement needed.

Here, the estimates of \( C_{20} \) variations from SLR, CSR, and JPL are compared. As the magnitude of estimates of \( C_{20} \) are small and the differences of the estimates from
different data centers are also small, the y axis cannot be distinguishable if plotting estimates of $\mathcal{C}_{20}$ or their differences directly. Here, values (estimates of $\mathcal{C}_{20}$ multiplied by $10^4$ and then added by 4.8417) indicating $\dot{\mathcal{C}}_{20}$ during 2002-2014 are shown in Figure 2.4. Obvious seasonal variations can be well observed from the time series of $\mathcal{C}_{20}$ estimates from SLR. The estimates of $\mathcal{C}_{20}$ from CSR and JPL show some differences from those of SLR, slightly larger in magnitudes and with more high frequency fluctuations. In Section 2.2.2, the derived global surface mass variations are shown using CSR RL05 monthly GRACE data in January 2006 without and with the replacement of SLR-$\mathcal{C}_{20}$. In Section 5.3, the effect of mass variations from $\mathcal{C}_{20}$ is discussed for the AIS mass variations.

![Figure 2.4 Time series of $\mathcal{C}_{20}$ estimates from CSR, JPL and SLR. Note: the $\mathcal{C}_{20}$ estimates are multiplied by $10^4$ and then added by 4.8417, for visual display purposes.]

2.2.2 Gaussian Smoothing

The temporal gravity information of the Earth obtained by GRACE observation is contaminated by satellite’s orbit error, K-band satellite-to-satellite ranging (KBR) measurements error, accelerometers error and so on. The most critical error is caused by the imperfect observation geometry of the KBR which is exclusively along-track with no observations in the radial and normal components, other than high-low GPS
tracking, which is at least a factor of 50 less accurate than KBR. This unobservability problem is the primary cause of the correlated error in the inverted gravity field solution at resonant spherical harmonics coefficient orders, which manifests in the so-called north-south stripes or high-frequency gravity errors. In order to reduce this error, Gaussian smoothing is commonly applied in order to minimize the north-south stripes, representing high-frequency errors present in the monthly mass variation derived from GRACE data. Gaussian averaging function $W_l$ was suggested by Jekeli (1981) to perform spatial average compensating for poorly determined short-wavelength spherical harmonic coefficients. It is defined to be only depending on the angle $\psi$ between two points $(\theta, \lambda)$ and $(\theta', \lambda')$ satisfying that \( \cos \psi = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\lambda - \lambda') \). Thus, the formula of computing the Earth’s mass redistribution in equivalent water height (EWH) of the Earth with Gaussian smoothing can be derived as follows (Wahr et al., 1998):

$$\vec{\Delta h}(\theta, \lambda) = \frac{2n a \rho_w}{3 \rho_w} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \frac{2l+1}{1+k_l} W_l \tilde{P}_{lm}(\cos \theta)(\Delta \tilde{C}_{lm} \cos(m\lambda) + \Delta \tilde{S}_{lm} \sin(m\lambda))$$  \hspace{1cm} (2.16)

where $W_l = \int_0^{\pi} W(\psi) P_l(\cos \psi) \sin \psi \, d\psi$, and $P_l$ are the Legendre polynomials satisfying that $P_l = \tilde{P}_{lm=0}/\sqrt{2l+1}$.

By normalizing the Gaussian average function so that the global integral of $W_l$ is 1 (Wahr et al., 1998):

$$W(\psi) = \frac{b}{2\pi} \exp[-b(1 - \cos \psi)] \left\{ \frac{1 - e^{-2b}}{1 - \cos(r/a)} \right\}$$  \hspace{1cm} (2.17)

where $r$ is the average radius which is the distance on the Earth’s surface at which $W_l$ reduces to half of its value at $\psi = 0$. In numerical computation, $W_l$ can be calculated using its recursion formula:

$$W_0 = \frac{1}{2\pi}$$

$$W_1 = \frac{1}{2\pi} \left[ \frac{1 + e^{-2b}}{1 - e^{-2b}} - \frac{1}{b} \right]$$

$$W_{l+1} = -\frac{2l + 1}{b} W_l + W_{l-1}$$  \hspace{1cm} (2.18)
By selecting different average radii, i.e., $r = 200 \text{ km}$, $250 \text{ km}$, $300 \text{ km}$, $350 \text{ km}$, $400 \text{ km}$, $500 \text{ km}$, $600 \text{ km}$, $800 \text{ km}$, the corresponding averaging function $W_l$ multiplying by $2\pi$ are shown separately from the left to the right in Figure 2.5. It can be seen that the larger the average radius, the slower is the decreasing of the corresponding averaging function $W_l$ multiplying by $2\pi$ versus the spherical degree. Conversely, the smaller average radius will correspond to the sharper decreasing of the averaging function.

As mention in Section 2.2.1, Figure 2.6 shows the surface mass variations derived from CSR RL05 monthly GRACE gravity field solutions after applying Gaussian smoothing with a radius of 300 km in January 2006 without and with the replacement of $C_{20}$ separately. Small differences can be observed, except for signals over the GrIS and AIS. In section 5.3, more details about the comparison of surface mass variations derived from GRACE data without and with the replacement of $C_{20}$ over the AIS can be found.

Figure 2.5 The plot of the averaging function $W_l$ versus spherical degree with different averaging radii: $r = 200 \text{ km}$, $250 \text{ km}$, $300 \text{ km}$, $350 \text{ km}$, $400 \text{ km}$, $500 \text{ km}$, $600 \text{ km}$, $800 \text{ km}$ (from the left to the right).
In addition, after applying Gaussian smoothing with different radii, i.e., 250 km, 300 km, 350 km, 400 km, 600 km, and 800 km, the surface mass variations derived from CSR monthly GRACE gravity field solution in January 2006 are separately shown in Figure 2.7. Obvious stripes can be seen in Figure 2.7(a). Applying Gaussian smoothing with a radius of 300 km, the stripes are removed over the polar region but few stripes are still observable in regions with low and intermediate latitudes, as shown in Figure 2.7(b). When the radius of Gaussian smoothing is 350 km, as provided by Figure 2.7(c), the surface mass variations can also be well observed over regions with low and intermediate latitudes. From Figure 2.7(d) to Figure 2.7(f), it can be seen that, as the radius of Gaussian smoothing increases, the magnitudes of signals in the land and polar region are reduced gradually. This means, the true signals can be attenuated after applying Gaussian smoothing with a relatively larger radius.

As the maximum spherical degree of the adopted GRACE gravity field solutions is 60, the spatial resolution of the surface mass variations derived from these GRACE gravity field solutions can be obtained by: \( R = \frac{\pi a}{l_{\text{max}}} = \frac{\pi a}{60} \approx 333 \text{ km} \). That is, a Gaussian smoothing with a radius of 333 km would be good to effectively minimize the north-south stripes. In this study, considering that the study region is polar ice sheets where not many stripes can be found after applying Gaussian smoothing with certain radius, and also taking account into the potential contamination from the signals over adjacent regions especially for the GrIS, as well as the potential attenuation of the original signals over both ice sheets, Gaussian smoothing with a radius of 300 km is applied for deriving surface mass variations from monthly GRACE gravity field solutions. It is also utilized during the post-processing of the outputs from SMB models and Envisat altimetry, in order to make the corresponding result commensurate with the spatial resolution of GRACE data.
Figure 2.7 Surface mass variations derived from CSR RL05 monthly GRACE gravity field solution in January 2006 (\(C_{20}\) replaced) after applying Gaussian smoothing with different radii: (a) 250 km, (b) 300 km, (c) 350 km, (d) 400 km, (e) 600 km, (f) 800 km.

2.2.3 Decorrelation

As the occurrence of the north-south stripes in the surface mass redistribution derived from monthly GRACE gravity field solutions, Swenson and Wahr (2006) first examined the spectral feature of the correlated errors. They found that some spherical harmonics with specific order from the monthly GRACE RL04 gravity field solutions are correlated. The geopotential coefficient orders are related to the resonances of the designed GRACE orbit, at order 14, 28, 42, 56, 70, and so on. By smoothing the harmonics for particular order with a quadratic polynomial in a moving window of width \(w\) centered at degree \(l\), they derived an empirical filter to isolate and remove varying coefficients of similar parity (For higher orders, coefficients of even and odd orders are not correlated with one another. But coefficients of even or odd orders are correlated with each other.).

The smoothed spherical harmonic coefficients can be written by:
\[ \Delta \tilde{C}_{lm}^s = \sum_{l=0}^{p} Q_{lm}^i l^i \]  

(2.19)

in which, \(Q_{lm}^i\) is the degree \(l\) coefficient corresponding to the fitted polynomial, \(p\) is the order of the polynomial. Similar formula can be given to \(\Delta S_{lm}^s\).

The polynomial coefficients \(Q_{lm}^i\) can be obtained by least squares fitting:

\[ Q_{lm}^i = \sum_{j=0}^{p} \sum_{n=1-w/2}^{l+w/2} n^{-1} L_{ij} \Delta \tilde{C}_{lm} \]  

(2.20)

where,

\[ L_{ij} = \sum_{n=1-w/2}^{l+w/2} n^i n^j \]  

(2.21)

It should be noted that the summation applies only to terms of the same parity as \(l\), i.e., \(l\) is odd, only odd degrees are then summed; \(l\) is even, only even degrees are then summed.

Thus, the smoothed spherical harmonic coefficient, \(\Delta \tilde{C}_{lm}^s\) can be derived from Eq. (2.19):

\[ \Delta \tilde{C}_{lm}^s = \sum_{l=0}^{p} \sum_{j=0}^{p} \sum_{n=1-w/2}^{l+w/2} L_{ij} \Delta \tilde{C}_{lm} l^i \]

\[ = \sum_{n=1-w/2}^{l+w/2} W_{lmn} \Delta \tilde{C}_{lm} \]  

(2.22)

in which, \(W_{lmn}\) is the filter defining by:

\[ W_{lmn} = \sum_{j=0}^{p} \sum_{j=0}^{p} L_{ij}^{-1} n^i n^j \]  

(2.23)

It can be seen that the correlated part of the monthly GRACE spherical harmonic coefficients is computed, and removed by fitting \(\Delta \tilde{C}_{l-2\alpha}^m, \Delta \tilde{C}_{l-2\alpha}^m, \Delta \tilde{C}_{l+2\alpha}^m, \cdots, \Delta \tilde{C}_{l+2\alpha}^m\), with \(\alpha = (w-1)/2\), using a quadratic polynomial. However, the window width \(w\) was not provided by Swenson and Wahr (2006).

Similarly empirical approaches were also developed by other studies (Chambers, 2006; Chen et al., 2007; Schrama et al., 2007; Wouters and Schrama, 2007; Kusche, 2007; Duan et al., 2009). In this study, no de-correlation technique is used over both polar ice sheets, the GrIS and AIS, as it has been noted that potential signal distortion in the spatial patterns of estimated trend or monthly solutions would
occur when decorrelations are applied to post-process GRACE monthly gravity field solutions.

2.2.4 Leakage Reduction

As shown in Figure 2.6, after applying Gaussian smoothing with certain radius, the signals of surface mass redistribution derived from GRACE data are attenuated in magnitude over the land area. Obviously, the original signals are leaked into oceans close to the coastlines, especially for both GrIS and AIS. This issue was considered by Wahr et al. (1998) who adopted an iterative algorithm to eliminate the effect of signal leakage. In previous studies, an empirical “scaling factor” was also applied by researchers to minimize the signal leakage when calculating regional mass changes (Velicogna and Wahr, 2006, 2013; Baur et al., 2009). Chen et al. (2006, 2013) used a forward modeling method to estimate mass change of the AIS, using observations from GRACE data.

In this study, we apply the leakage reduction technique from Guo et al. (2010). As mentioned by Guo et al. (2010), this leakage reduction method can avoid side lobes effect resulting from directly smoothing GRACE mass change data regionally (Wahr et al., 1998). It assumes that the north-south stripes and high-degree noises are effectively minimized by applying Gaussian smoothing or de-correlation technique. It also assumes that the signal of mass variation derived from monthly GRACE gravity field solution over oceans after removing the portion provided by GRACE non-tidal high-frequency atmospheric and oceanic mass variation models routinely generated at GFZ as Atmosphere and Ocean De-aliasing Level-1B (AOD1B) products is quite small. Thus, compared with the smoothed mass variation on the land, the contribution from signals over oceans close to the coastlines can be neglected. The leaked signals can be recovered by re-scaling the Gaussian-smoothed mass changes so that the result after the leakage reduction is equal to that obtained by a weighted averaging of land signals.

The Gaussian function can be rewritten by:

\[ G(\psi) = \exp \left( -\frac{\psi^2}{2\sigma_\psi^2} \right) \]  

(2.24)

where \( \sigma_\psi \) can be derived from the smoothing radius \( r_\psi \):

\[ \sigma_\psi = \frac{r_\psi}{\sqrt{2\ln(2)}} \]  

(2.25)

For the classical isotropic Gaussian filter, the smoothed value of \( \Delta h(\theta, \lambda) \) obtained in Eq. (2.14) at the point \((\theta', \lambda')\) is defined by:

\[ \Delta h(\theta', \lambda') = Y^{-1} \int_0^\pi d\theta \int_0^{2\pi} \Delta h(\theta, \lambda) G(\psi) \sin \theta \, d\lambda \]  

(2.26)
where

\[ Y = \int_0^\pi d\theta \int_0^{2\pi} G(\psi)\sin \theta \, d\lambda \]  \hspace{1cm} (2.27)

That is, \( Y \) is independent on the point \((\theta', \lambda')\).

Eq. (2.26) can be written in discrete form in numerical calculation:

\[ \overline{\Delta h}(\theta'_j, \lambda'_k) = Y^{-1}(\theta'_j, \lambda') \sum_{(\theta_\alpha, \lambda_\beta) \in S} \Delta h(\theta_\alpha, \lambda_\beta) G(\psi_{jk}^{\alpha\beta}) \sin \theta_\alpha \]  \hspace{1cm} (2.28)

\[ Y(\theta'_j, \lambda'_k) = \sum_{(\theta_\alpha, \lambda_\beta) \in S} G(\psi_{jk}^{\alpha\beta}) \sin \theta_\alpha \]  \hspace{1cm} (2.29)

where

\[ S = \{ L, \text{ for } (\theta, \lambda) \text{ is over the land}, \}

\[ O, \text{ for } (\theta, \lambda) \text{ is over oceans} \]

so as to distinguish whether the location of the grid cell \((\theta_\alpha, \lambda_\beta)\) is over the land or ocean. \((\theta'_j, \lambda'_k)\) and \((\theta_\alpha, \lambda_\beta)\) are grid points with sampling interval \((\Delta \theta, \Delta \lambda)\):

\[ \theta_j = (j + 0.5)\Delta \theta, j = 0, \cdots, \pi/\Delta \theta - 1; \]

\[ \lambda_k = k\Delta \lambda, k = 0, \cdots, 2\pi/\Delta \lambda - 1; \]  \hspace{1cm} (2.30)

\[ \theta_\alpha = (\alpha + 0.5)(\Delta \theta/\mu), \alpha = 0, \cdots, \mu(\pi/\Delta \theta) - 1; \]

\[ \lambda_\beta = \beta(\Delta \lambda/\mu), \beta = 0, \cdots, \mu(2\pi/\Delta \lambda) - 1; \]  \hspace{1cm} (2.31)

in which, \( \mu \) is an odd integer which is applied to subdivide the sampling interval. In order to make sure the original grid cell centers are the same centers of certain subdivided grid cells, \( \mu \) should be odd. In this study, \( \mu = 3 \) is used.

Suppose that the point \((\theta'_j, \lambda'_k)\) is on the land, the Eq. (2.28) and Eq. (2.29) can be rewritten separately:

\[ \overline{\Delta h}_L(\theta'_j, \lambda'_k) = Y^{-1}(\theta'_j, \lambda') \sum_{(\theta_\alpha, \lambda_\beta) \in L} h(\theta_\alpha, \lambda_\beta) G(\psi_{jk}^{\alpha\beta}) \sin \theta_\alpha \]  \hspace{1cm} (2.32)

\[ Y_L(\theta'_j, \lambda'_k) = \sum_{(\theta_\alpha, \lambda_\beta) \in L} G(\psi_{jk}^{\alpha\beta}) \sin \theta_\alpha \]  \hspace{1cm} (2.33)

Based on the assumption that the original signal over the oceans is small enough so that its contribution to the summation in Eq. (2.28) is negligible, the following approximation can be written:

\[ \sum_{(\theta_\alpha, \lambda_\beta) \in S} h(\theta_\alpha, \lambda_\beta) G(\psi_{jk}^{\alpha\beta}) \sin \theta_\alpha \approx \sum_{(\theta_\alpha, \lambda_\beta) \in L} h(\theta_\alpha, \lambda_\beta) G(\psi_{jk}^{\alpha\beta}) \sin \theta_\alpha \]  \hspace{1cm} (2.34)
Thus, the final smoothed mass variations over the land after correcting the leakage reduction can be derived:

\[
\bar{\Delta h}_L(\theta_j', \lambda_k') = \frac{\Delta h(\theta_j', \lambda_k')}{\bar{y}(\theta_j', \lambda_j')}
\]

(2.35)

In this study, the above leakage reduction method is used to correct the GRACE mass variations over the GrIS and the AIS. The same leakage reduction approach with the same mask (used to distinguish the land and ocean) used for GRACE data is applied to outputs from RACMO2 models to correct the surface mass variations over both ice sheets. The same leakage reduction approach with a different mask is adopted for correcting elevation changes derived from Envisat altimetry after Gaussian smoothing in order to make the result be commensurate with the spatial resolution of GRACE data.
Chapter 3: Satellite Radar Altimetry Data Processing

3.1 Radar Altimetry Basics

The principal objective of satellite radar altimetry is to measure the range from antenna phase center onboard the satellite to the sea surface, which in simple terms means that an onboard altimeter transmits a short pulse of microwave radiation towards the ground at nadir and the two-way time delay of the echo is measured. Thus, the range $R$ of the altimeter above the surface can be computed as (Chelton et al., 2001):

$$R = \frac{ct}{2} - \sum_j \Delta R_j$$  \hspace{1cm} (3.1)

where $c$ is the propagation velocity of electromagnetic waves in vacuum, and $\Delta R_j$, $j = 1, \cdots$ are corrections for the various components of atmospheric refraction and biases between the mean electromagnetic scattering surface and mean sea level at the air-sea interface.

Using a specified reference ellipsoid defined by two parameters, which could be the best fitting ellipsoid to the Earth’s geometric shape, the range measurement is then converted to the height $h$ of the surface relative to the reference ellipsoid by (Chelton et al., 2001):

$$h = H - R = H - \frac{ct}{2} + \sum_j \Delta R_j$$ \hspace{1cm} (3.2)

in which, $H$ is the height of the satellite relative to the reference ellipsoid.

For the radar altimeter, considering the size of the antenna and the accuracy of measurements due to antenna pointing errors, the pulse-limited mode of operation is preferred on spacecraft, as shown in Figure 3.1 (Rapley, 1990). Radar pulse with a duration of $\tau$ is transmitted from the antenna to the Earth’s surface, and parts of them will be reflected after the interaction with the surface. In order to make sure that the echo reflected from the Earth’s surface can be sensed by the altimeter (the noise of the receiver), the Signal-to-Noise Ratio (SNR) needs to be certain appropriate value. However, such a SNR for the short pulse challenges the satellite power system, as a quite high transmit power will be needed; it also makes the lifetime of the transmitter limited (Fu and Cazenave, 2001). Thus, a technique called pulse compression is stated by Ulaby.
et al. (1981) to get a frequency-modulated long pulse transmitted. More details can be found in Chelton et al. (1989). Then, the received echo is mixed with a deramping chirp which is the same as the transmitted chirp but the corresponding frequency is offset by an intermediate frequency. Such a processing is called full deramping. Furthermore, a fast Fourier transform is applied to analyze the deramped signal (Rapley, 1990) to obtain spectral estimates at corresponding discrete frequencies (so-called “gates”). The above processing concerning the received echo is shown in Figure 3.2. As provided in Figure 3.2(d), the returned power distribution can be regarded as a function of gates or time is so-called the waveform (Brown, 1977). The half-power point at the leading edge of waveforms is thought to be corresponding to the two-way travel time. The altimeter only provides measurement over a range window (typically 60 m). As the distance between the satellite and the surface varies along the orbit of the satellite, an onboard tracker is used to align the half-power point at the track point fixed at certain gate (i.e., for Envisat Ku band, the total number of gates is 128 and the nominal track point is at 46.5), and predict the position of next echo, based on the information obtained from the echo just recorded by the receiver. Thus, the reflected signal from the surface corresponding to the next pulse can fall in the range window for the altimeter. In this way, the altimeter can provide continuous measurements over relatively flat surface.

Figure 3.1 Pulse-limited altimeter operation (modified from Rapley, 1990).
Figure 3.2 Diagram for the conversion of the received radar echo to a waveform. (a) echos from the reflecting elements on the surface, (b) replica of transmitted chirp, (c) deramping chirps organized in an order of frequency, (d) the truncated power waveform in time domain (adopted from Scharroo 2002).

Figure 3.3 illustrates 3 stages of the interaction between a return echo with a duration of $\tau$ for a pulse-limited altimeter, and a monochromatic sea surface with wave height $H_w$ (Chelton et al., Chapter 2, Fu and Cazenave (2001) is the whole book). When the radar pulse firstly arrives the ground, the area with shape like an expanding cycle will be illuminated (epoch $t_0$), at the same time, the echo reflected from the sea surface begins to return to the altimeter. When the two-way travel time $t$ satisfies the condition, $t_0 < t < t_1$, the area of the pulse-limited footprint on the sea surface grow continuously with time, until the trailing edge of the pulse arrives at the sea surface at nadir direction (epoch $t_1$). After that, the shape of the pulse-limited footprint will be like an expanding annulus with constant surface area (epoch $t_2$) (Chelton et al., Chapter 2, Fu and Cazenave (2001) is the whole book).
Figure 3.3 Geometrical description for the interaction of a pulse of duration $\tau$ with the scattering surface in 3 stages corresponding to epoch $t_0$, $t_1$, and $t_2$ (Modified from Chelton et al., Chapter 2, Fu and Cazenave (2001) is the whole book).

The illuminated region on the surface will contribute to the radar return measured by the pulse-limited radar. In this paragraph, we show the derivation of the calculation of the radius and corresponding area of pulse-limited footprint Chelton et al., Chapter 2, Fu and Cazenave (2001) is the whole book. As indicated in Figure 3.3, the distance between the antenna and the planar surface is $R_0$. That is, the reflection of the leading edge of the pulse from the planar surface at nadir arrives at the altimeter at two-way travel time $t_0(H_w) = 2R_0/c$. At a two-way travel time $t_0 < t < t_1$, the altimeter measures the reflection of the leading edge of the pulse from wave crests along the outer perimeter of the illuminated region at off-nadir angle $\theta_{out}$ and corresponding radial distance $r_{out}$ from the nadir point. Define the slant range to the planar surface of wave crests at off-nadir angle $\theta_{out}$ to be $R_{out} = R_0 + \Delta R_{out}$, and neglect the curvature of the Earth’s surface, and then,

$$r_{out}(t, H_w) = \sqrt{R_{out}^2 - R_0^2} = \sqrt{2R_0 \Delta R_{out} + \Delta R_{out}^2}$$  \hspace{1cm} (3.3)

Since $\Delta R_{out} \ll R_0$, the following approximation can be obtained:

$$r_{out}(t, H_w) \approx \sqrt{2R_0 \Delta R_{out}}$$  \hspace{1cm} (3.4)
According to Chelton et al. (1989), the corrected footprint radius on a spherical Earth with a radius of $R_e$ is:

$$r_{out}(t, H_w) \approx \frac{2R_0\Delta R_{out}}{1+R_0/R_e}$$  \hspace{1cm} (3.5)

As the elapsed time between the arrival time $t_0$ of the first reflection from the wave crest and the time $t$ of the leading edge of the pulse reflected from wave crests at off-nadir angle $\theta_{out}$ can be written as:

$$\Delta t(H_w) \equiv t - t_0 = \frac{2\Delta R_{out}}{c}$$  \hspace{1cm} (3.6)

Substituting Eq. (3.6) into Eq. (3.5),

$$r_{out}(\Delta t, H_w) = \frac{c\Delta t R_0}{1+R_0/R_e}$$  \hspace{1cm} (3.7)

The corresponding area of the footprint can be computed:

$$A_{out}(\Delta t, H_w) = \pi r_{out}^2 = \frac{\pi c\Delta t R_0}{1+R_0/R_e}$$  \hspace{1cm} (3.8)

At a two-way travel time $t > t_1$, the illuminated area becomes an annulus. Here we don’t derive the formulation for calculating the radius of the inner perimeters and corresponding area, and more details can be found in Chelton et al., Chapter 2, Fu and Cazenave (2001).

In addition to the illuminated area on the surface, the antenna gain pattern and the normalized radar cross section of the surface will also contribute to the returned power measured by an altimeter. The time series of the average returned power, i.e., waveform $W(t)$, received from the surface in the near-nadir direction, can be given by the following convolution model (Brown, 1977; Hayne, 1980; Barrick and Lipa, 1985; Rodriguez, 1988), assuming that the reflected surface is deep ocean:

$$W(t) = P_S(t) * s_r(t) * q_s(t)$$  \hspace{1cm} (3.9)

where $P_S(t)$ is the average surface impulse response on a spherical earth, $s_r(t)$ is the radar system point-target response, $q_s(t)$ is related to the surface elevation probability density of scattering elements, and $t$ is the measured time at the satellite with $t = 0$ corresponding to the first arrival time of a reflected echo from the ocean surface.

Assuming that the altimeter antenna pattern can be given by a Gaussian approximation, and taking the effect from the Earth’s curvature into account, the radar impulse response
from a smooth sphere can be written (Hayne, 1980; Barrick and Lipa, 1985; Rodriguez, 1988):

\[ P_5(t) = A \exp(-\alpha t) I_0(\beta t^{1/2}) U(t) \] (3.10)

in which

\[ \alpha = \frac{\ln 4}{\sin^2(\theta/2)} \frac{c}{h(1+h/R)} \cos(2\xi) \] (3.11)

\[ \beta = \frac{\ln 4}{\sin^2(\theta/2)} \left[ \frac{c}{h(1+h/R)} \right]^{1/2} \sin(2\xi) \] (3.12)

where \( A \) is a constant which depends on the radar parameters, the ocean reflectivity and the off-nadir angle, \( I_0(\beta t^{1/2}) \) is a modified Bessel function, \( U(t) \) is a unit step function, \( c \) is the speed of light, \( h \) is the altimeter height above the mean ocean surface, \( R \) is the radius of the Earth, \( \xi \) is the off-nadir pointing angle, \( \theta \) is the antenna beamwidth.

In Eq. (3.9), \( q_s(t) \) is the surface specular point density function expressed in radar return time domain. The formula corresponding to \( q_s(t) \) expressed in spatial domain can be written as follows:

\[ f_{sp}(z) = \frac{1}{(2\pi \sigma^2)^{1/2}} \exp\left(\frac{-z^2}{2}\right) \left\{ 1 + \frac{4}{6} (\zeta^3 - 3\zeta) \right\} \] (3.13)

\[ \zeta = \frac{(z-z_0+\gamma z/2)}{\sigma} \] (3.14)

where \( z \) is the height above the mean sea surface, with \( z = 0 \) indicating the mean sea surface, \( \sigma \) is the ocean standard deviation, \( \lambda \) is the ocean surface skewness, \( \gamma \) is the cause of the electromagnetic bias, \( z_0 \) is the tracker error mainly explaining the altimeter height estimation error. Using a change of variables \( t = -2z/c \), Eq. (3.13) can be converted to the radar return time domain.

In Eq. (3.9), \( s_r(t) \) is the radar system point-target response (PTR), which primarily indicates the transmitted radar pulse shape but also the effects of the bandwidth of the receiver. According to Rodriguez (1988), the PTR can be given:

\[ s_r(t) \sim \frac{\sin^2[(at/2)(T-t)]}{(at/2)^2}, \quad -T < t < T \] (3.14)

where \( 2T \) is the radar pulse length, and \( a \) is a constant which depends on the radar bandwidth.
3.2 Waveform Retracking Methods

As mentioned in 3.1, the onboard tracker will keep the waveforms well centered in range or power in the analysis window. An onboard algorithm is then applied to these waveforms to derive the ocean surface parameters which are used to generate the real time operational sensor data record (OSDR) products (Desai and Vincent, 2003). As limited by the speed of computation, very simple onboard algorithms are used without estimating the ocean surface skewness and at the cost of degrading the accuracy and reducing the number of the estimated parameters (Barrick and Lipa, 1985). In order to obtain the most accurate parameters, ground retracking is usually performed, i.e., the developed maximum likelihood estimator (MLE) algorithm with 3 or 4 parameters applied to waveforms from Jason-1/Jason-2 over inland water surface (Amarouche et al., 2004). Besides, for a reasonable flat land or ice surface, the altimeter can maintain onboard tracking, i.e., altimeters such as those on the ERS-1/2 and Envisat satellites designed to operate in ice mode. However, range measurements over these surfaces can be difficult to interpret because the leading edge of the waveform may present the return from the closest topographic feature away from the nadir point (Fu and Cazenave, 2001). In such case, ground-based waveform processing with special models for the waveform shape (waveform retracking) is necessary to obtain accurate range estimate. Retracking is primarily consisted of computing the departure of the leading edge position of each waveform from the tracking point and correcting the range measurement. Many retracking algorithms are developed to retrieve heights over ice sheet surface (Martin et al., 1983; Wingham et al., 1986; Davis, 1995; Davis, 1996; Davis, 1997). Here, 4 retrackers, ocean retracker, the NASA/Goddard Space Flight Center (GSFC) Version 4 retracker, Offset Center of Gravity (OCOG) retracker, and threshold retracker are described.

3.2.1 Ocean Retracker

As mentioned in Section 3.1, the mean return altimeter waveform can be expressed as the convolution shown in Eq. (3.9) which can be written like a matrix equation \( y = Mx \), with \( y \) represents the return power, \( M \) is the convolution \( P_s(t) * s_r(t) \), and \( x \) is the specular point probability density function. Considering the presence of noise and the singularity of Eq. (3.9), Rodriguez (1988) proved that for off-nadir angles less than or of the same order as the antenna beam width, the so called corrected slopes convolution model can be derived as follows:

\[
P_s(t) = A \exp(-\alpha t)I_0(\beta t^{1/2})U(t) = A \exp\left[-\left(\alpha - \frac{\beta^2}{4}\right)t\right]U(t) \tag{3.15}
\]

Thus, the derivative of the return power can be written:

\[
W'(t) = [s_r(t) * q_s(t)] * P_s'(t)
\]
Then,
\[ s_r(t) * q_s(t) = \frac{1}{A} [W'(t) + \left( \alpha - \frac{\beta_x^2}{4} \right) W(t)] \]  
(3.17)

When implementing the corrected slopes convolution model, the derivative can be approximated by:
\[ W'(t) = \frac{W(t+\Delta t) - W(t-\Delta t)}{2\Delta t} \]  
(3.18)

where \( \Delta t \) is the sampling interval.

Using the least squares method, the corresponding parameters can be solved:
\[ x = [M^T M]^{-1} M^T y \]  
(3.19)

3.2.2 The GSFC Version 4 Retracker

Due to the complex feature of ice sheet topography and the slow response of the altimeter onboard tracker, the leading edge ramp of the waveform frequently departed from the tracking location. Thus, retrackers were provided by Martin et al. (1983) to compute the departure of the leading edge ramp of the waveform from the tracking point, and to correct the range measurements over ice sheet surface. They included two models corresponding to two different types of waveform shapes, with the model including 5 parameters for waveform with one ramp, and the model including 9 parameters for waveform with two ramps. Based on these models, the ice altimetry group of NASA/GSFC and Raytheon STX developed the Version 4 retracker for waveforms from the ERS-1 satellite altimetric data. Here, the two models are described below. The single ramp altimeter waveform model is given:
\[ y = \beta_1 + \beta_2 e^{-\beta_3 t} P \left( \frac{t-\beta_4}{\beta_4} \right) \]  
(3.20)

where \( Q_1 = \begin{cases} 0; & t < \beta_3 - 2\beta_4 \\ (t - (\beta_3 - 2\beta_4)); & t \geq \beta_3 - 2\beta_4 \end{cases} \), and \( P(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-q^2/2} \, dq \)

The retracted range obtained from this model is:
\[ R = R_{obs} + \Delta R \]
\[ = R_{obs} + (\beta_3 - \text{tracking gate}) \Delta s \]  
(3.21)

with \( \Delta s = c t/2 \), where \( R_{obs} \) is the indicated range telemetered from the spacecraft, \( \Delta R \) is the retracking range correction, \( \beta_3 \) is in units of waveform gates, tracking gate is the...
location in gates of the altimeter track point, \( c \) is the speed of light, and \( \tau \) is the duration of the transmitted pulse.

The double ramp altimeter waveform model is as follows:

\[
y = \beta_1 + \beta_2 (1 + \beta_9 Q_1) P \left( \frac{t - \beta_3}{\beta_4} \right) + \beta_5 e^{-\beta_6 Q_2} P \left( \frac{t - \beta_6}{\beta_7} \right)
\]

(3.22)

Where \( Q_1 = \begin{cases} 0; & \text{for } t < \beta_3 + 0.5\beta_4 \\ t - (\beta_3 + 0.5\beta_4); & \text{for } \beta_6 - 2\beta_7 > t \geq \beta_3 + 0.5\beta_4 \\ \beta_6 - 2\beta_7; & \text{for } t \geq \beta_6 - 2\beta_7 \end{cases} \)

\( Q_2 = \begin{cases} 0; & \text{for } t < \beta_6 - 2\beta_7 \\ (t - (\beta_6 - 2\beta_7)); & \text{for } t \geq \beta_6 - 2\beta_7 \end{cases} \), and \( P(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-q^2/2} \, dq \)

For this model, \( \beta_3 \) and \( \beta_6 \) are the locations in gates of the centers of the first and second ramps, respectively. The retracted ranges are then:

\[
R_1 = R_{\text{obs}} + \Delta R_1 = R_{\text{obs}} + (\beta_3 - \text{tracking gate})\Delta s
\]

(3.23)

\[
R_2 = R_{\text{obs}} + \Delta R_2 = R_{\text{obs}} + (\beta_6 - \text{tracking gate})\Delta s
\]

(3.24)

3.2.3 OCOG Retracker

The OCOG retracking algorithm, which is a purely statistical approach, which does not depend on any model, was developed by Wingham et al. (1986) to perform robust retracking. It calculates the centre of gravity (COG) and width (W) of a rectangular box from the waveform data, as provided in Figure 3.4. The amplitude (A) of the box is twice the height of the COG. The corresponding formulation is like the following:

\[
A = \sqrt{\frac{\sum_{i=1+n_1}^{N-n_2} P_i^4(t)}{\sum_{i=1+n_1}^{N-n_2} P_i^2(t)}}
\]

(3.25)

\[
W = \left( \frac{\sum_{i=1+n_1}^{N-n_2} P_i^2(t)}{\sum_{i=1+n_1}^{N-n_2} P_i^4(t)} \right)^2 / \left( \sum_{i=1+n_1}^{N-n_2} P_i^2(t) \right)
\]

(3.26)

\[
COG = \frac{\sum_{i=1+n_1}^{N-n_2} iP_i^2(t)}{\sum_{i=1+n_1}^{N-n_2} P_i^2(t)}
\]

(3.27)

Where, \( P_i \) is the waveform power, \( N \) is the number of waveform samples, \( n_1 \) and \( n_2 \) are the numbers of bins affected by aliasing at the beginning and end of the waveform, respectively. Thus, the leading edge position (LEP) can be given by:

\[
LEP = COG - \frac{W}{2}
\]

(3.28)
The OCOG retracking algorithm is sometimes used to compute the initial values for the Threshold retracker (Bamber, 1994). It is also the algorithm behind the ice-1 retracker for Envisat RA-2 altimeter, optimized for general continental ice sheets.

3.2.4 Threshold Retracker

Over the continental ice sheets, it is known that both surface and volume scattering affect the shape of the altimeter return waveform, particularly over dry snow regions. However, the algorithm developed by Martin et al. (1983) is based solely upon a surface scattering model, and the OCOG algorithm by Wingham et al. (1986) is a model-free retracker. Davis (1993) developed the retracking algorithm to take into account the contribution from both surface and volume scattering to the waveform. Davis (1995) compared the elevation change retrieved by the threshold retracking algorithm with that from other algorithms such as NASA’s 5 or 9 parameter algorithm, and ESA’s OCOG retracker. Davis (1996) discussed threshold retracking algorithm with 3 different threshold levels, 10%, 20%, and 50% of the amplitude given by the OCOG method, respectively. He pointed out that the 10% level is only appropriate in regions of ice sheet dominated by volume scattering, i.e., the interior of the ice sheet; the 50% level is appropriate in regions where surface scattering dominated the return echo; the 20% level can be used for areas of ice sheets where combinations of surface and volume scattering contribute to the shape of the waveforms. The steps and formulas for the method are given by (Vignudelli et al., 2011):

The thermal noise is calculated firstly:

\[ P_N = \frac{1}{5} \sum_{i=1}^{5} p_i \]  

(3.29)
where $P_N$ is the average value of the power in the first five gates, and $p_i$ is the power in gate $i$.

The threshold level is then computed:

$$T_h = P_N + q \cdot (A - P_N)$$

where $q$ is the threshold value, i.e., 10%, or 20%, or 50%, $A$ is calculated in Eq. (3.25), $T_h$ is the power corresponding to the threshold value $q$.

The power at the retracked gate on the leading edge of the waveform is computed as:

$$G_r = G_{k-1} + \frac{T_h - p_{k-1}}{p_k - p_{k-1}}$$

Where $G_k$ is the power at the $k$th gate, and $k$ is the location of the first gate exceeding $T_h$.

3.3 Surface Gradient Correction

For radar altimetry over continental ice sheets, two different approaches, the crossover analysis and along-track repeat analysis are commonly used. In the crossover analysis, measurements are used at crossing points of the satellites’ orbit ground tracks, with the elevation differences computed (Wingham et al., 1998; Davis and Ferguson, 2004). As the location of measured heights from the ascending and descending tracks at the crossing point are the same, the effects from surface topography on both heights are common (Yang et al., 2014). By differencing, the surface slope effects can be largely reduced. However, the crossover analysis has its own limitations, i.e., the relative sparse measurements, as indicated by blue dots shown in Figure 3.5. To be noted, the red lines in Figure 3.5 depict all the observations. The crossover analysis is also limited by tracker biases from ascending and descending passes with anisotropic altimeter antenna. In order to fully use the observations from altimetry, along-track repeat analysis was developed (Legresy et al., 2006). As the ground tracks are not exactly repeated deviating approximately ±1 km from the nominal track, the observations are affected by surface gradient. Generally speaking, two methods, use of DEM and modified collinear analysis, are adopted to correct the surface gradient effect. More details about the two methods for correcting the surface gradient effect are provided in the following.
3.3.1 Use of DEM Method

Using an external DEM for the ice sheets, the heights at the spot of observation $Q$ and the corresponding nominal track $P$ can be computed separately by bi-linear interpolation. The difference between both heights from the DEM can be regarded as an approximation of the height difference between the observed height along the actual track and the height at the nominal track. Thus, the height at the nominal track $P$ can be calculated (Yi, 1995):

$$h_{pn} = h_{Qo} + (DEM_{Pn} - DEM_{Qo})$$

(3.32)

where $h_{Qo}$ is the height observed from altimetry at an actual track $Q$, $DEM_{Qo}$ and $DEM_{Pn}$ are heights at an actual track $Q$ and at nominal track $P$ separately from the DEM, and $h_{pn}$ is the height at the nominal track $P$. Repeating the procedure, the time series of heights can be formed at each nominal track, except for those tracks where at least one of the three heights, $h_{Qo}, DEM_{Qo}$ and $DEM_{Pn}$ is not available.

Su et al. (2015) used this method to generate time series of heights over the GrIS, using the DEM generated from the first seven campaigns of ICESat data (DiMarzio et al., 2007). Example studies concerning use of DEM approach to generate ice sheet elevation time series can be found in Slobbe et al. (2008) and Sorensen et al. (2011). To be mentioned, no ICESat data is analyzed in this study because of the limitation from temporal sampling (2-3 month per year).
3.3.2 Modified Collinear Method

As the coverage and accuracy of DEM are relatively limited, here we modify the along-track repeat analysis based on collinear analysis using the data from altimetry, avoiding potential error contaminated by the poor quality of DEM, and mitigating the different retracker biases from ascending and descending tracks caused by unsymmetrical altimeter antenna onboard of Envisat. Similar method can be found in Legresy et al. (2006), Ewert et al. (2012), and Flament and Remy (2012).

Along each reference track, we select a box with a size of 350 m × 2 km (along-track sampling distance × cross-track repeat band) and centered at the corresponding reference track. Within each of the boxes, a mathematical model used to fit all observations can be written by:

\[
R_i = R_0 + a_1 x_i + a_2 y_i + a_3 x_i^2 + a_4 x_i y_i + a_5 y_i^2 + a_6 x_i^2 y_i + a_7 x_i y_i^2 + a_8 x_i^2 y_i^2 + c \sin\left(\frac{2\pi t_i}{T}\right) + d \cos\left(\frac{2\pi t_i}{T}\right)
\]

(3.33)

where \(i\) is the counter \((i = 1, 2, 3, \ldots, N)\). \(N\) is the number of measurements within the box. \(t_i\) indicates the time relative to the reference epoch, 2003. \(x_i, y_i\) are the along track and cross track coordinates respectively within the local plane. \(T\) is the period of seasonal signal, only considering \(T = 1\) year. \(R_i\) is the observed elevation from Envisat altimetry. \(H, a_0, a_1, \ldots, a_8, c, d\) are unknown parameters to be estimated. Apparently, three components are included in the mathematical model given by Eq. (3.33). The first one is a linear time-dependent parameter \(H\), representing the trend of elevation change. The second is a quadratic polynomial, which approximates the local surface topography. The third is a seasonal term including a sine and a cosine term. Note, \(H\) does not indicate the true trend of elevation change, because \(H\) is contaminated by the component related to the properties of snow surface and the upper firn layers.

From Eq. (3.33), there are 12 parameters to be estimated, thus at least 13 observations are needed, as redundant observations are always recommended in the least squares adjustment. Reference tracks with observations less than 13 in the box are discarded. That is, \(N\) larger than 12 becomes one criterion of data editing. We then perform least squares adjustment, and remove the topographic gradient effect to form time series of elevation change at each reference track. Furthermore, the residual generated by observations subtracting the fitted values and its root mean square (RMS) values are calculated. Following the criterion of \(3\sigma\) editing, we remove the measurements whose corresponding residuals in absolute values are larger than three times of the RMS. If the number of the remaining observations is larger than 12, the least squares adjustment is performed again, and the residual and its RMS are also calculated in an iterative procedure. If RMS at this time is larger than 5 m, we then discard this measurement entirely. Along the reference track, by repeating the above process, the time series of
elevation change can be generated at the reference tracks with observations of good quality. Example methods of using planar surface as a local reference surface to correct the surface gradient effect can also be found in studies by Howat et al. (2008), Smith et al. (2009), and Sorensen et al. (2011).

3.4 Correction from Backscattered Power and Waveform Shape Parameters

As the change of snow roughness and upper firn layers may cause artificial elevation change, backscatter coefficients, which are sensitive to changes of surface or subsurface properties, are used to correct the elevation change (Wingham et al., 1998; Davis and Ferguson, 2004). A linear relationship is empirically assumed between the temporal changes in height and backscatter coefficient. The components of height variations linearly related to changes on backscatter coefficients are removed. If \( dH \) indicates the time series of elevation change after surface gradient correction, \( N \) is the number of observed elevation change, and \( d\sigma_0 \) is the produced time series of backscatter coefficient change, the \( i \)th \( (i = 1, \ldots, N) \) elevation change after correcting the effect from backscattered power can be written by:

\[
(dH_{c})_i = (dH)_i - CG \cdot (d\sigma_0)_i
\]  

(3.34)

where \( (dH_{c})_i \) is the \( i \)th elevation change after applying the correction from backscatter coefficient, \( CG \) is the correlation gradient obtained by linear regression.

As pointed out by Remy et al. (2012), using the data retrieved by the ice-2 retracking algorithm, which is optimized for ocean-like radar echoes over the continental ice sheets (Legresy and Remy, 1997), the difference in the trends of elevation changes corrected with different empirical corrections exceeds 5 cm/yr in some place over the AIS. They concluded that a correction with only the backscatter is not enough and waveform parameters, the leading edge width and the second part of trailing edge slope, are needed. A linear relationship between elevation change and changes of the leading edge width, the second part of trailing edge slope, backscatter coefficient, are assumed only for the ice-2 retracking algorithm. More details can be found in Legresy et al. (2005) and Flament and Remy (2012). For the ice-1 algorithm, only backscatter coefficient is computed, and it does not have waveform shape parameters such as the leading edge width and the second part of trailing edge slope. Thus, only corrections from backscatter coefficient can be applied for the ice-1 retracking algorithm.

As to modified collinear method, for the ice-1 and the ice-2 algorithms, besides generating elevation change, when retaining individual observations within the box, we also preserve its corresponding parameters like backscatter coefficient and waveform shape parameters. Using the same procedure as generating elevation change, we form time series of backscatter changes, and changes of waveform shape parameters. Thus, the time tags for these time series are exactly the same as that for corresponding elevation change at the same location.
Chapter 4 : Greenland Ice Sheet Mass and Elevation Changes

4.1 Introduction

The GrIS is the largest continental ice in the northern hemisphere, with an area of $1.736 \times 10^6$ km$^2$ and a volume of $2.6 \times 10^6$ km$^3$, containing about 10 percent of the Earth’s fresh water (Benn and Evans, 2010). According to Williams and Ferrigno (2009), if the GrIS completely melts and distributes evenly across the world’s ocean, it can cause a sea-level rise of 6.5 m. Having experienced the increase of runoff, surface melting due to increased summer temperature (Krabill et al., 2004; Hall et al., 2008; van den Broeke et al., 2009), and ice discharge from accelerating outlet glaciers (Rignot and Kanagaratnam, 2006; Howat et al., 2007; Enderlin et al., 2014), the melting rate of the GrIS appears to have accelerated during the past decade. Arguably decadal or longer variations over the ice sheet could be mistaken as an acceleration signal. Published values of the trend estimates differ widely, as provided in Table 4.1, and thus have large uncertainties, most probably due to the significant influence by inter-annual (Sasgen et al., 2012) or longer variations since the time span of data utilized by these studies are relatively short.

<table>
<thead>
<tr>
<th>Author</th>
<th>Method/sensor</th>
<th>GRACE solution</th>
<th>Period</th>
<th>Mass [Gt/yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zwally et al. (2005)</td>
<td>ERS1/2</td>
<td></td>
<td>04/92-10/02</td>
<td>-139±68</td>
</tr>
<tr>
<td>Chen et al. (2006)</td>
<td>GRACE</td>
<td>CSR</td>
<td>04/02-11/05</td>
<td>-219±21</td>
</tr>
<tr>
<td>Ramillien et al. (2006)</td>
<td>GRACE</td>
<td>CNES</td>
<td>07/02-03/05</td>
<td>-119±10</td>
</tr>
<tr>
<td>Luthcke et al. (2006)</td>
<td>GRACE</td>
<td>Mascon</td>
<td>01/03-12/05</td>
<td>-101±16</td>
</tr>
<tr>
<td>Wouters et al. (2008)</td>
<td>GRACE</td>
<td>CSR</td>
<td>02/03-01/08</td>
<td>-179±25</td>
</tr>
<tr>
<td>Velicogna (2009)</td>
<td>GRACE</td>
<td>CSR</td>
<td>04/02-02/09</td>
<td>-230±33</td>
</tr>
<tr>
<td>van den Broeke et al. (2009)</td>
<td>Mass budget</td>
<td></td>
<td>01/03-12/08</td>
<td>-237±20</td>
</tr>
<tr>
<td>Slobbe et al. (2009)</td>
<td>GRACE</td>
<td>CSR</td>
<td>04/02-06/07</td>
<td>-218±18</td>
</tr>
<tr>
<td>Slobbe et al. (2009)</td>
<td>GRACE</td>
<td>GFZ</td>
<td>08/02-06/07</td>
<td>-168±05</td>
</tr>
<tr>
<td>Ewert et al. (2012)</td>
<td>GRACE</td>
<td>GFZ</td>
<td>08/02-06/09</td>
<td>-191±21</td>
</tr>
</tbody>
</table>

Table 4.1 The GrIS mass change trend estimates based on GRACE and radar altimetry sensors and various methods, and covering different periods.
Three approaches are commonly used to estimate the trend of the GrIS mass loss. The mass budget approach partitions the individual components of ice sheet mass balance, including SMB and ice discharge. It is susceptible to small errors present in the input from accumulation and output by ice flow and melting. Rignot and Kanagaratnam (2006) utilized satellite radar interferometry observations to measure glacier velocities in Greenland. Combining with SMB (obtained by modelling different surface processes, i.e., snowfall accumulation and run off), they reported a total GrIS loss of $201.6 \pm 36.9$ Gt ($-224 \pm 41$ km$^3$/yr) in 2005, with 61% contribution from ice discharge anomalies. However, van den Broeke et al. (2009) concluded that SMB and ice discharge anomalies contributed equally to the GrIS mass loss, using the monthly output of the RACMO2/GR and ice flux data from 38 glacier drainage basins. They mentioned that the shift to 50% can be fully explained by the larger inter-annual variability and stronger downward trend in their updated SMB time series.

Satellite gravimetry enables us to infer the mass variations of ice sheets utilizing measurements of temporal gravity changes observed by the GRACE twin satellites since 2002. As shown in Table 4.1, there are several recent investigations concentrating on the trend of mass variations in the GrIS using GRACE data. But the results vary within a range of more than 100 Gt/yr, depending on the GRACE data products used, the time period covered and the algorithm adopted. Actually, the data span up to the present time (2015) is only ~10 years, the estimates of the trend from GRACE are significantly influenced by inter-annual or longer variations. Furthermore, the spatial resolution of GRACE-derived mass variations is relatively coarse.

The third approach, satellite radar altimetry, measures the elevation changes of ice sheet. Thus, the volume changes of ice sheets can be readily inferred. With an appropriate density assumption, the volume changes of ice sheets can be converted into mass changes. Different density assumptions are adopted by previous studies, e.g., Zwally et al. (2005) used a density of 900 kg m$^{-3}$ for solid ice and 400 kg m$^{-3}$ for the upper part of the firn. Thomas et al. (2006) and Slobbe et al. (2009) assumed a density of $600 \pm 300$ kg m$^{-3}$ for the interior and a density of 900 kg m$^{-3}$ for regions close to ocean so as to convert elevation changes to mass changes. That is, the knowledge of surface density is a large uncertainty hindering the estimate of mass changes derived from satellite altimetry. This problem could get worse as elevation changes occur in the interior of ice sheet, especially in the case of smaller elevation change over a larger area.

In this chapter, we apply the SMB outputs from two different versions of the RACMO2/GR model and compare them in three components, trend, seasonal, and inter-annual components. Similarly, we also analyze the total mass variations in those three components, and compare the mass variations derived from the mass budget method and from GRACE gravimetry. Furthermore, using Envisat data from January 2003 to December 2009, we derive the elevation change in the GrIS, applying two different methods to correct surface gradient. Finally, the inter-annual anomalies of mass change
from GRACE, elevation change from Envisat, and surface mass change from SMB models are combined to analyze the nominal density over the GrIS.

4.2 Surface Mass Balance of the GrIS

SMB of the GrIS is defined as the sum of accumulation by precipitation with two individual components of snowfall and rainfall, and ablation by sublimation and runoff (van den Broeke et al., 2009). SMB subtracting ice discharge (D) across the grounding line can be used to obtain mass balance (MB). Thus, the above relationship can be given as follows:

\[
MB = \frac{\partial M}{\partial t} = SMB - D
\]

\[
SMB = \text{snow} + \text{rain} - \text{sublimation} - \text{runoff}
\]

Here, monthly outputs of SMB from the RACMO2/GR model in two different versions, v2.1 and v2.3, with a horizontal resolution of about 11 km, are analyzed. The period of outputs from the model in v2.1 is from January 1960 to December 2012, and that from the model in v2.3 is from January 1960 to December 2013. As mentioned in previous studies, the mean field of SMB during the period 1961-1990 can be used as a climatological mean since the GrIS was close to balance during these years (Rignot et al., 2008). Similar to previous studies, and to assess that monthly outputs from the RACMO2/GR model in both versions, the cumulative SMB anomalies referring to the 1961-1990 mean are computed separately in Greenland, as plotted in Figure 4.1. Compared with the result provided by van den Broeke et al. (2009) in their Figure 2(A), by Sasgen et al. (2012) in their Figure A.2(A), and by Vernon et al. (2013) in their Figure 5, the two cumulative SMB anomaly curves in Figure 4.1 are quite similar to those shown in previous studies, although it is expressed in mass in EWH not in Gt. Given an approximate area of \(1741 \times 10^3 \text{ km}^2\) for the whole GrIS, the SMB cumulative anomaly is about -2000Gt at the end of year 2012. In addition, it is clear that the cumulative anomaly from the model in v2.3 is less negative than that in v2.1, especially after the year 2000. This could be associated with the upgrade of RACMO2/GR using schemes of drifting snow and grain size-dependent albedo (van Angelen et al., 2014).
Figure 4.1 Cumulative SMB anomalies from RACMO2/GR in two different versions, v2.1 (red dotted line) and v2.3 (blue dotted line), referring mean field during 1961-1990.

Furthermore, by dividing the mask of the GrIS into 7 basins similar to basins used in Sasgen et al. (2012), as shown in Figure 4.2, the SMB cumulative anomaly is computed in each basin. Figure 4.3 depicts the monthly SMB cumulative anomaly for each basin. Here, the SMB variations are computed in Gt in each basin, which are the same as SMB cumulative anomaly in mass in Gt in each basin published by Sasgen et al. (2012). Compared to Figure A.2 in Sasgen et al. (2012), the SMB cumulative anomaly in each basin is visibly similar to their results. The small discrepancies can be caused by the differences in the masks used for each basin. To some extent, this confirms our processing of the monthly outputs from the RACMO2/GR model is indeed correct. On the other hand, the differences of the SMB cumulative anomalies from the SMB model in two different versions can be seen; over basin a, cumulative SMB anomalies from the model in both versions are very similar to each other, and apparent decreasing can be seen after year 2000; slight differences are shown in basin f and g; relatively large differences are depicted in other 4 regions. That is, over the North and West GrIS, the upgrade of RACMO2/GR model shows slight differences, while it makes noticeable differences on the Northeast, East, and South GrIS.
Figure 4.2 Mask for 7 basins in the GrIS (Each letter from a ~ g in white color indicates one basin, and each basin is shown in small color squares with the same color.).
Figure 4.3 Cumulative SMB anomalies from RACMO2/GR model in v2.1 (red dotted line) and v2.3 (blue dotted line) for the GrIS (referenced to the mean field during 1961-1990).
Furthermore, in order to better understand the features and differences of SMB variations in both models, we decompose the SMB variations into three components, trend, seasonal and inter-annual variations. In order to be commensurate with GRACE result, a Gaussian smoothing with a radius of 300 km is applied before the decomposition. As Gaussian smoothing may cause signals to leak into oceans close to coastline, leakage reduction is conducted according to Guo et al., (2010), using a mask shown in Figure 4.4.

Figure 4.4 Masks used for leakage reduction in the SMB, and GRACE (a), as well as Envisat (b) data processing.

The model we use to fit the time series of SMB variations can be written by:

\[ m(t) = a_0 + a_1 t + a_2 \sin(\omega_1 t) + a_3 \cos(\omega_1 t) + a_4 \sin(\omega_2 t) + a_5 \cos(\omega_2 t) \]  

(4.3)

where, \( t \) is the time in year relative to year 2003, \( a_0, \cdots a_5 \) are unknown parameters, \( m(t) \) is the time series of SMB variations at a regular grid, \( \omega_1 \) and \( \omega_2 \) are the frequencies corresponds to the periods of one year and half year, respectively.

From Eq. (4.3), it is obvious that \( a_1 \) indicates the trend. In order to quantify seasonal variation, we simply define the amplitude \( Amp \) and phase \( \phi \) like this:

\[ Amp = \sqrt{a_2^2 + a_3^2} \]  

(4.4)

\[ \phi = \tan^{-1}\frac{a_3}{a_4} \]  

(4.5)

in which, if \( \phi \in [0, \pi] \), \( 2\pi - (\phi - \frac{\pi}{2}) \) indicates the epoch corresponding to the maximum of seasonal amplitude; if \( \phi \in [-\pi, 0] \), \( -2\pi - (\phi - \frac{\pi}{2}) \) indicates the corresponding epoch for peak value of seasonal amplitude.
The inter-annual anomaly at each grid point is computed according to the following approach: (a) Fit time series based on least squares with an offset, a linear trend, and the two sinusoids representing seasonal variations; one has a period of a year, and the other half a year, i.e., the formulation in Eq. (4.3). (b) Remove the linear trend and seasonal variations from the time series. (c) Since only the average seasonal variations in the time span of data have been removed in the previous step, while seasonal variations may be different from year to year, a moving average with a one year long window is applied to further remove the remnant seasonal variations.

Figure 4.5 depicts trends from both models during the period from January 2003 to December 2012, showing that mass is losing largely (<-5 cm/yr in EWH) along coastal regions in the North, West, South, and Southeast GrIS and that mass increases in a quite small magnitude in the interior. From Figure 4.5, less negative mass loss can be seen from the model in v2.3, especially in the North, Northwest, and Southeast GrIS. Figure 4.6 maps the corresponding errors in the trends from both models. The errors are small, comparing with the magnitudes of the trends. After the upgrade of SMB model, the errors of trend are reduced, especially over coastlines in the Northwest and South GrIS.

Figure 4.5 Trends from the RACMO2/GR model in (a) v2.1 and (b) v2.3.
Figure 4.6 Errors of trends from the RACMO2/GR models in (a) v2.1 and (b) v2.3.

Figure 4.7 shows the amplitudes of seasonal component of SMB variations over the GrIS. It is obvious that, for both models, the amplitude of seasonal change is relatively smaller in the interior than that in coastal regions in the GrIS. From Figure 4.7(b), the amplitude of seasonal change is almost less than 5 cm in the interior, while it is about 5 cm – 15 cm over the coastal regions. The largest amplitudes of seasonal change from both models are in the South GrIS, with amplitudes larger than 25 cm from the model in the old version, and of about 15 cm from the model in the new version. That is, the amplitudes of seasonal changes are reduced after the upgrade of the SMB model. Figure 4.8 maps the phase of seasonal variations from both models. From Figure 4.8(a), phases with a range of 90-150° can be seen in the West GrIS, and phase in other regions are around 60°. From Figure 4.8(b), similar patterns of phases can be observed but with different magnitudes, i.e., for the region in the West GrIS with a latitude around 75°, the phase is about 60°, while for other regions, the phase is less than 30°.

Figure 4.9 shows the inter-annual anomalies of SMB from both models over the GrIS during years 2003-2012. Similar patterns can be visibly observed during most of years, except for noticeable differences shown in the Northeast GrIS in 2005, Southeast GrIS in 2007. The inter-annual variations with relatively large amplitudes ranging from 10 cm to 20 cm primarily appears over the Northwest, West, Southwest, and Southeast Greenland, with mass losing in years 2003 and 2004, increasing from 2005 to 2007, and then decreasing from 2008 to 2012. Over the Southwest Greenland, inter-annual mass variation increases in 2009. In the interior of the GrIS, the amplitude of inter-annual variations from SMB models is relatively small, i.e., around 5 cm.
Figure 4.7 Amplitudes of seasonal terms from RACMO2/GR models in (a) v2.1 and (b) v2.3.

Figure 4.8 Phase (in Degree) of seasonal variations from RACMO2/GR models in (a) v2.1 and (b) v2.3.
Figure 4.9 Inter-annual anomalies from RACMO2/GR models in v2.1 (v1) and v2.3 (v3).
4.3 GRACE-derived Mass Variations in the GrIS

We use GRACE RL05 monthly gravity field solutions from three data centers, CSR, GFZ, and JPL, during the period from January 2003 to December 2013. Also, the monthly gravity field solutions with the same time periods generated by the Ohio State University (OSU) are also analyzed over the Greenland (Shang et al., 2015). Each solution of monthly gravity field includes spherical harmonic coefficients with degree and order up to 60. Degree-2 and order 0 \( C_{20} \) Stokes coefficients are replaced with satellite laser ranging estimates. As it is known that spurious north-south stripes as high frequency errors appear in the mass changes derived from GRACE monthly gravity solutions (Swenson and Wahr, 2006; Duan et al., 2009). We apply the Gaussian smoothing with a radius of 300 km to reduce the stripes in the surface mass variation. No decorrelation is conducted, alleviating the potential signal distortion in the spatial patterns of the interannual or longer variations over the GrIS. Gaussian smoothing causes leakage of mass change signal into the ocean close to coastline. Here leakage reduction is performed according to Guo et al. (2010), using the extended mask by Duan (2013) shown in his Figure 5.3. Thus, the monthly mass changes in EWH are computed at the regular grids with latitude interval of 0.5° and longitude interval of 1°, according to the formulation provided by Wahr et al. (1998), forming a time series of mass change at each grid point. Figure 4.10 shows three time series of mass variations derived separately from GRACE data from 3 data centers over Greenland during the span from January 2003 to December 2013. Obviously, significant and consistent mass loss can be observed from all the time series of mass variations. To be mentioned, interpolation is used when no good observation from GRACE is available. Thus, for the products from the three data centers, there is no data gap during the epoch: 2003/06, 2011/01, 2011/06, 2012/05, 2012/10, 2013/03, 2013/08, and 2013/09.

The estimates of the trends of mass variations in the Greenland are -267±5.5 Gt/yr from CSR, -264.5±6.6 Gt/yr from GFZ, and 251.8±5.8 Gt/yr from JPL. The results are consistent with those shown in Figure 4.10, with the most negative trend from CSR, and less negative trend from GFZ. These estimates are close to the reported values by Flechtner et al. (2013) during 2003-2013, especially for the result, -269.1±2.5 Gt/yr, from CSR products. The slight discrepancies may be caused by different data processing.
In order to investigate the characteristics of total mass variations and compare them with the features from SMB variations in three components, in the following, the mass variations are derived in trend, seasonal and inter-annual variations in Greenland using CSR RL05 monthly GRACE gravity field solutions during the period from January 2003 to December 2012. Figure 4.11 maps the trend and its error obtained from the fitting using Eq. (4.3) in Greenland. Significant mass loss with amplitude larger than 10 cm in EWH can be seen primarily over the Northwest, West, Southwest, South, and Southeast Greenland. Slightly mass gain is in the interior. Comparing Figure 4.11(a) with Figure 4.6, similar pattern can be observed, i.e., the location of significant mass loss. It also can be found that the pattern of trend from RACMO2/GR model in v2.3 is much closer to that revealed by GRACE data, although no component of dynamic mass loss is added to SMB variation. Figure 4.11(b) depicts the corresponding error which is small (less than 1 cm/yr) with respect to the trend derived from GRACE data, similar to the error of trend shown in Figure 4.6(b) for the trend of SMB variation.
Figure 4.11 Trend (a) of mass variations and its error (b) (obtained during the fitting) in Greenland derived from CSR monthly GRACE RL05 data products during 2003-2012.

Figure 4.12 shows the amplitude and phase of seasonal variations in Greenland during 2003-2012. Obviously, the seasonal amplitude in the North is much smaller than that in the south, with the smallest seasonal amplitude of less than 2 cm in a circular region in the West and the largest amplitude of around 20 cm in the South Greenland. This is similar to the pattern of the seasonal amplitude of SMB variations shown in Figure 4.7(a). For seasonal phase, there is a small region with a phase range of 0°–30° in the interior of the Greenland. For coastal regions, the seasonal phase is around –60°~–20°, except for a small region with phase ranging less than –60° in the West Greenland.
Figure 4.12 Amplitude (a) and phase (b) of seasonal variations in Greenland derived from CSR monthly GRACE RL05 data products during 2003-2012.

Figure 4.13 depicts the inter-annual anomalies derived from CSR monthly GRACE RL05 data products during 2003-2012 in Greenland. The spatial pattern of inter-annual anomalies is significant, with mass loss in years 2003, 2004 over the Northwest, West, and South Greenland, mass gain from year 2005 to year 2010, and then mass loss from 2011 to 2012. The inter-annual anomalies in the interior are relative small, i.e., less than 5 cm. Compared with the inter-annual SMB variations in Figure 4.9, very similar pattern can be found during the whole study period. It can be observed that the inter-annual SMB variations from RACMO2/GR v2.1 are more consistent with these inter-annual mass variations from GRACE data. However, it cannot be taken for granted that SMB model in v2.1 is better than its upgrade. It should be kept in mind that mass variation observed by GRACE is the sum of SMB variation and mass change through ice flow.
4.4 Mass Change by Mass Budget Method and GRACE in Greenland

Since the last decade, surface processes including snow, rain, melting, refreezing, and runoff can be well captured over the whole GrIS, with a spatial scale about 11 km (van den Broeke et al., 2009). This enables researchers to estimate the mass variations in Greenland using mass budget method. Sasgen et al. (2012) compared the sum of SMB variations from RACMO2/GR (v2.1) and ice discharge estimated based on Interferometric Synthetic Aperture Radar (InSAR) satellite observations of ice flow within 38 drainage basins with mass variations from GRACE data over the whole Greenland and 7 divided basins. Their results indicate that over 4 basins, the trends derived from mass budget method exceed that from GRACE data; for 2 basins, the trends from both approaches are close to each other; for 1 basin in the West, trend from GRACE data is significantly larger than that from mass budget method. van Angelen et al. (2014) evaluated outputs from RACMO2 (it may be v2.3, not mentioned) with weather stations and GRACE data, and they pointed out that signals with higher amplitudes are observed by GRACE, i.e., more snow accumulation in winter and more meltwater runoff in summer. Enderlin et al. (2014) updated the estimate of ice discharge and combined it with SMB variations from van Angelen et al. (2014) to provide an improved mass budget in Greenland.
Here we use SMB from RACMO2/GR in v2.3 and the annual ice discharge during years 2003–2012 from Enderlin et al. (2014) to calculate cumulative annual mass change, and compare it with the total mass variation from GRACE, as provided in Figure 4.14. It can be seen that the mass variation from mass budget method is consistent with that from GRACE data, and it provides a lower boundary for the observed mass variation by GRACE. Figure 4.15 shows the fraction of the differences between mass variations from mass budget method and GRACE data relative to mass variations by mass budget method. The residual of mass variations accounts for approximately less than 15% of the mass variation. From 2003 to 2007, the mass variations from mass budget method and GRACE data are well consistent with each other. After year 2007, their differences increase gradually, and after year 2010, the fraction is reduced gradually.

Figure 4.14 Time series of cumulative annual mass change from mass budget method and GRACE data. Blue line indicates SMB, yellow line for ice discharge, cyan line for mass variation combining SMB and ice discharge, and red line for mass variation by GRACE data, as well as red shading for corresponding error bar for mass variation estimated using GRACE data.
4.5 Elevation Changes from Envisat Altimetry

Satellite altimetry provides geometric observations about ice sheets. Using Envisat data during 2003-2009, the characteristics of elevation change of the GrIS are investigated (we select this time period, mainly considering the poor quality of data at the beginning of the mission and that the extended mission has a different repeat orbit). As mentioned in Section 3.3, two different approaches are used separately to correct the effect from surface topography to form corresponding time series of elevation changes from Envisat data during 2003-2009. In the first approach, use of DEM, the GLAS/ICESat Laser Altimetry DEM of Greenland with the grid spacing of 1 km (DiMarzio, 2007), is used to perform the surface gradient correction. Figure 4.16 shows the statistics of RMS before and after the surface gradient correction, using an external DEM. After the surface gradient correction, the percentage of data with RMS of elevation change of less than 2 m increases significantly. With the RMS of elevation change of less than 5 m, the percentage of data is about 90%, which to some extent confirms that the topographic effect can be removed by the use of DEM method.
As the second approach, the box with a size of 350 m×2 km (along-track sampling distance × cross-track repeat band) is chosen at each reference track. A bi-quadratic model is then used to fit the observations, and the time series of elevation change are formed at each reference track after removing the topographic effect. As mentioned in Section 3.3, at least 13 observations are needed. Here, the numbers of observation within each logic box is shown in Figure 4.17. Over most regions, the number of observations within the logic box is larger than 60; for a few passes, the number of observations within the box is around 50; over coastal regions, especially at some sparse points in the North and Southeast Greenland, the number of observations within the box is less than 40. This illustrates the bi-quadratic surface can be well fitted using these observations.
Furthermore, the RMS of elevation changes within each box before the surface gradient correction is computed, and that for the time series of elevation change generated at each reference track after the surface gradient correction is also calculated, as shown in Figure 4.18. Obviously, the topographic effect is well removed from the original elevation changes. With RMS less than 2 m, the percentage of valid data is around 90%. And with RMS less than 5 m, the percentage of valid data is around 99%. These confirm that the effectiveness of removing topographic effect from modified collinear method is much better than that from the use of DEM method. In addition, the percentage of data with RMS in smaller amplitudes in Figure 4.18(a) is larger than that in Figure 4.16(a). This is because the ground tracks are limited in the along-track direction in a relatively smaller region in Figure 4.16(a).
Figure 4.18 Statistics of RMS of height change time series before (a) and after (b) removing the topographic effect, using modified collinear method.

For the correlation correction, the correlation maps between elevation change and changes on backscattered power from both approaches are shown separately in figure 4.19. Highly positive correlations can be observed primarily in the interior of the GrIS, and weakly negative correlations can be found mostly over coastal regions (close to Jakobshavn Glacier) in Southwest, Northeast, and parts of Southeast GrIS. Significant differences can be seen between the two correlation maps; highly positive correlation coefficients ($>0.6$) over much wider regions in the interior of the GrIS are shown from modified collinear method; relatively weakly positive correlations can be observed in the interior after removing topographic effect using the external DEM; over a small region in the Northeast GrIS, Figure 4.19(a) shows more negative correlations but Figure 4.19(b) shows more positive correlations on the order of 0.5 and less negative correlations.
Figure 4.19 Correlation maps between elevation change and change on the backscattered power after correcting surface gradient by the use of DEM (a) and modified collinear method (b).

Similar to the investigation done for SMB and total mass variations from GRACE, the generated time series of elevation changes are decomposed into three components. Figure 4.20 shows the trends in high spatial resolution from both methods. Similar features can be observed: elevation is reducing mostly along the coastal regions in the Greenland, including the Northwest, West, Southwest, and Southeast as well as part of Northeast GrIS; slightly elevation increase can be seen along the central ridge; the significant elevation increase is observed at South Greenland with a latitude around 66°N, which is consistent with that shown in a previous study in their Figure 1 (McConnell et al., 2000). Besides, a much cleaner map of trend of elevation change is shown by modified collinear method.
Figure 4.20 Trends of elevation change after correcting surface gradient by the use of DEM (a) and modified collinear method (b).

Figure 4.21 depicts the corresponding uncertainties of the trends shown in Figure 4.20. It can be seen that uncertainties of the trend of elevation change from the DEM method are small in the interior but relatively large over the coastal regions in the GrIS; for modified collinear method, uncertainties of the trends are much smaller over wider region in the interior, and large error occurs only in the edge of the GrIS.
Figure 4.21 Uncertainties obtained during the fitting for the corresponding trends of elevation change after correcting surface gradient by use of DEM (a) and modified collinear method (b).

Figure 4.22 maps the amplitude of seasonal variations during 2003-2009 from both methods. In the interior of the GrIS, the seasonal amplitude is relatively small (~10 cm) from both methods. Over the coastal regions, especially along the West and Southeast GrIS, the seasonal amplitude is much larger (> 50 cm), which is associated with the high accumulation there (van den Broeke et al., 2011). Comparing with the result from the DEM approach, the seasonal amplitude from modified collinear method shows relatively smaller magnitude in the wider regions in the interior, and smaller region with large seasonal amplitude along coastal regions.
Figure 4.22 Seasonal amplitude of elevation change after correcting surface gradient by the use of DEM (a) and modified collinear method (b).

Figure 4.23 depicts the phase of seasonal variation of elevation change from both methods over the GrIS during the same time period. Both maps are noisy but they generally and consistently show that the phase of seasonal variation in the North is relatively larger (~120°–150°) than that (~−150°–−120°) in the South GrIS. There are small regions with a phase ranging 120° to 150° in the South. Negative phase can be observed primarily along the West and Southeast coastline.
The above features of elevation change in three components are shown in high spatial resolution. In order to know whether the features are similar to those from SMB in Section 4.2 and GRACE data in Section 4.3, a Gaussian smoothing with the same radius is applied to the above result from Envisat altimetry. For the leakage reduction, considering the coverage of Envisat data, the smaller mask shown in Figure 4.4(b) is used. Figure 4.24 – Figure 4.27 show the corresponding results after the smoothing. Similar patterns can be observed in the corresponding figures from SMB variation, GRACE data, and Envisat altimetry. Here no detailed description are repeated.
Figure 4.24 Spatially smoothed trends of elevation changes from using the DEM (a) and the modified collinear (b).

Figure 4.25 Errors of the corresponding trends from the DEM (a) and the modified collinear (b) method after smoothing.
Figure 4.26 Seasonal amplitude of elevation change from the DEM (a) and the modified collinear (b) method after smoothing.

Figure 4.27 Phase of seasonal variations of elevation change from the DEM (a) and the modified collinear (b) method after smoothing.

Here the inter-annual anomalies are not shown, which will be discussed in the next section in detail.
4.6 High Inter-annual Anomalies Combining GRACE and Envisat Altimetry

4.6.1 The Inter-annual Anomalies and Their Correlation

As mentioned in section 4.5, the inter-annual time series of elevation change from both methods in the GrIS are extracted from Envisat during 2003-2009. In order to compare the inter-annual anomalies of elevation change from Envisat altimetry and mass change from GRACE, monthly CSR RL05 GRACE gravity field solutions during 2003-2009 are analyzed using the same method described in section 4.3, and to be commensurate with spatial resolution of the GRACE data, the same Gaussian smoothing is applied to the two inter-annual time series of elevation change from Envisat from the DEM and modified collinear method separately. Figure 4.28 shows the inter-annual anomalies extracted from GRACE and Envisat datasets in year 2003-2009 after subtracting the corresponding mean field during the whole study period. Amplitude of larger than 10 cm can be found in the inter-annual signals derived from both independent datasets (GRACE data in EWH and Envisat data in height), especially along the coastal region. In the interior of the GrIS, the amplitude is slightly smaller. Comparing the top figures (from GRACE dataset) with the two panels at the bottom (from Envisat dataset), similar patterns can be observed for most features, for instance, in the year 2003, 2004, 2007, and 2009, although there are some differences in these two datasets. For GRACE data, signals with large amplitudes in the region close to coastline like Northwest GrIS might be contaminated by leakage error since there are islands in the West close to the GrIS. For the two inter-annual time series obtained using two different methods from Envisat, similar patterns with slight differences can be found during most years but significant difference can be seen in some years, i.e., year 2005, signals with much larger amplitudes can be found in the inter-annual anomalies by modified collinear method. Over the South Greenland, the amplitude of the inter-annual anomalies by modified collinear method is generally larger than that obtained from the DEM method, while signals with slightly larger amplitude can be seen in the Northwest GrIS from the DEM method. These differences are primarily associated with different processing about the surface gradient correction.
Figure 4.28 Inter-annual anomalies of mass change in EWH from GRACE (a-g) and height changes from Envisat (RA) using the DEM (h-n) and modified collinear method (o-u) separately in the GrIS during years 2003-2009.

To assess the similarity of these two inter-annual time series, the correlation between them is calculated over regions covered by both datasets in the GrIS, with the result shown in Figure 4.29(a). From the figure, we find that the inter-annual anomalies derived from both datasets are highly positively correlated in North (N), Northwest (NW), Southwest (SW), and a small part of Northeast (NE) and East (E) Greenland. Generally speaking, taking the central ice divide indicated by the black line in Figure 4.29(a) in Greenland as a boundary, the inter-annual anomalies from these two datasets show good agreement in the west side. A total of 60% of the regions including a small part of East GrIS show correlation coefficients larger than 0.7, which suggests that pronounced inter-annual anomaly resulting from the common geophysical source are detected by these two independent techniques. In contrast, in most of Southeast and a small part of Northeast GrIS, lower correlation values ranging from –0.6 to 0.4 are found. The percentage for the regions with negative correlation coefficients is about 7%.

Here we show the statistics in detail within 9 regions with strong positive, moderate positive as well as negative correlation. These regions are indicated by blue rectangles numbered 1 to 9 in Figure 4.29(a). The distributions of RMS of elevation changes are
shown in Figure 4.30(a) – (i). Under the data editing criterion of retaining data points with RMS less than 5 m, we discard about 10% of the measurements for each region. It appears that the RMS of elevation change are slightly larger in region 4, 5, 6, 7, and 9, but no clue indicates poor qualities of elevation change time series in region 7 and 9.

We also calculate the correlation between the two inter-annual variations derived from Envisat dataset under a more strict data editing criterion, retaining the elevation change time series with the RMS of less than 2.5 m, and GRACE products, with result shown in Figure 4.31(a). However, the negative correlation is still apparent in region 9, while the previous weak negative correlation in Southeast GrIS becomes weakly positive. We should mention that this correlation map in Figure 4.31(a) may not reflect the true scenario because it is at the expense of deleting almost 50% data (See Figure 4.16(b)).

In addition, we compute the correlation between inter-annual time series extracted from the RACMO2/GR, which is a SMB model (It is an independent source and the outputs in two different versions, v2.1 and v2.3, are used.), and GRACE, with result shown in Figure 4.32. We find that for both SMB models, the correlation is almost positive over the whole GrIS except for one spot in Southeast part in Figure 4.32(b); high correlation with magnitude larger than 0.7 are found over most of the GrIS; weakly positive correlation appears in Southeast and a small part of region 9 in the GrIS. This suggests that, what GRACE measures is the surface mass variation in the GrIS. To some extent, it confirms the consistency between inter-annual SMB variations and those obtained from GRACE, indicating that the inter-annual elevation changes derived from Envisat reflect the inter-annual surface mass variation in the GrIS except for the two regions of negative correlation. Using the inter-annual time series of GRACE derived mass variations and simulated snow depth from ECMWF reanalysis, Bergmann et al. (2012) depicted, in their Figure 6, a very similar correlation pattern over GrIS, with a negative correlation in Southeast and a small part of Northeast Greenland, including our two regions of negative correlation. The significant difference is that our two negatively correlated regions are separated by a small part of positively correlated region in East GrIS. On the other hand, by comparing the correlation map with the bed topography (Bamber et al., 2001; Layberry and Bamber 2001), as shown in Figure 4.29(b), we find that the negative correlations occur in regions where bedrock elevation is high, although there exists no evidence that bedrock elevation may influence the correlations of inter-annual anomalies of either mass and surface elevation changes. We notice that basal melting was occurring in region 9 as shown in Figure 2 of Fahnestock et al. (2001). Whether basal melting is associated with inter-annual anomalies or not is unknown.
Figure 4.29 Comparison between the correlation map and bedrock topography. (a) Correlation between the inter-annual variations from GRACE and Envisat during the span from 2003 to 2009. (b) Bedrock topography (Bamber et al., 2001; Layberry and Bamber 2001).
Figure 4.30 Histogram of the RMS of surface height over 9 regions.

The correlation map between the inter-annual anomalies of mass change from GRACE and elevation change using modified collinear method is shown in Figure 4.31(b). Similar
to the correlation map in Figure 4.29(a), correlation coefficient on the order of 0.7 is found primarily over parts of the Northeast, West, Southwest, and part of Southeast GrIS; negative correlation occurs in a smaller area in Region 9. Significant difference is that weakly positive correlation occurs in part of Southeast, Northeast, and Northwest GrIS. For the Southeast and Northwest part, the correlation coefficient is more like the case shown in Figure 4.31(a).

Figure 4.31 Correlation of inter-annual time series from GRACE and Envisat data with RMS of elevation change (<2.5 m) as an editing criterion (a), GRACE and Envisat data using modified collinear method (b) during 2003-2009.
4.6.2 Nominal Density associated with Inter-annual Anomalies

As inter-annual variations can be well observed by both techniques, i.e., mass variations by GRACE and volume changes by Envisat, the combination of these two inter-annual time series enable us to derive a nominal density of the portion related to the inter-annual mass changes. Here assuming that the inter-annual firn compaction component, which will alter the elevation change but not mass change, is small, we neglect this term. The elevation change observed by Envisat is distributed throughout the firn or ice column, thus, there is no associated physical layer in the firn with the nominal density to be estimated.

Figure 4.33 shows the inter-annual anomalies of mass change in EWH from GRACE and height change from Envisat using both methods in 9 regions in the GrIS, with (1)-(9) indicating the corresponding region number in Figure 4.29(a). From Figure 4.33, we find that the inter-annual anomalies from GRACE (blue dotted line) and Envisat (red dotted line for the DEM method and green dotted line for modified collinear method) are consistent well with each other except for region 7 and 9. The amplitude of inter-annual anomalies in each region is about 5-10 cm, except for slightly large amplitude shown in
region 6, 7, and 8 by modified collinear method. In region 9, the amplitude of inter-annual mass variations from GRACE is relatively smaller, 3 cm; the amplitudes of the two inter-annual elevation variations from Envisat are about 5 cm – 10 cm; the two inter-annual elevation changes show significant differences, with red dotted line indicating a periodic signal (~2 years) and green dotted line showing longer variations. It also can be seen that the patterns of inter-annual anomalies in region 1-6 are different from these in region 7-9, which indicates the spatial variability of inter-annual anomalies along the central ice divide in the GrIS. For the DEM method, in region 2, 5, and region 6, the inter-annual anomalies from GRACE and Envisat show slightly better consistency. This indicates an agreement with the correlation shown in Figure 4.29(a), i.e., for regions with higher positive correlation coefficients, the inter-annual variations from both datasets show more similar patterns. For modified collinear method, in region 4 and 5, better consistency can be seen from Figure 4.33; however, significant differences in the amplitudes of signals can be observed in region 1 and 6 where highly positive correlation can be seen from Figure 4.31(b); this may be associated with different density values for corresponding mass and elevation change. Using the two SMB models, inter-annual surface mass variations are extracted in each region, as shown in cyan dotted and pink dotted lines in Figure 4.33. Similar patterns can be seen between inter-annual anomalies from SMB models and GRACE data.

Furthermore, by combining the inter-annual anomalies of mass change and the two elevation changes separately, the average surface densities are estimated separately in each region by linear regression, using the following formula:

\[ \rho_w h_w + p = \rho_a h_E^t + b \]  \hspace{1cm} (4.6)

in which, \( \rho_w \) is the density of water, \( h_w \) is the GRACE-derived inter-annual mass change in EWH during time period \( t \), \( h_E^t \) is the Envisat-observed inter-annual elevation change during the same period, \( \rho_a \) is the unknown nominal density, and \( b \) is the unknown bias.

As provided in Table 4.2, for the DEM method, the density values for seven selected regions of positive correlation range from 383.7±50.9 to 596.2±34.1 kg m\(^{-3}\), and the corresponding density values associating with modified collinear method range from 299.15±11.2 kg m\(^{-3}\) to 764.57±43.4 kg m\(^{-3}\). These estimated densities are close to or larger than the density of snow and less than the density of pure ice. This suggests that, for both methods, all the estimated nominal density values over regions of positive correlation follow the fact that the surface of ice sheets consists of snow, firn, and ice. As modified collinear method is more effective in removing surface gradient effect, the estimated densities from this method is more reliable. From Table 4.2, we also notice that the nominal density in region 1-3 in the North GrIS is slightly smaller, comparing with other regions of positive correlation in the West, which is in agreement with the results of Zwally et al., (2011), in Figure 7c, that the nominal density obtained from firm compaction model is generally smaller (e.g. < 500 kg m\(^{-3}\)) in the northern part of the GrIS. Besides, the uncertainties in Table 4.2 indicate the corresponding uncertainties
during the linear regression. This may be associated with the magnitude of correlation coefficients listed in Table 2. For region 7 and 9, the nominal densities by both methods are not in a physically feasible range.

Figure 4.33. Inter-annual variations from GRACE (blue dotted line) in EWH, Envisat (red dotted line by use of DEM; green dotted line by modified collinear method) in height, SMB models in v2.1 (cyan dotted line) and v2.3 (pink dotted line) in nine selected regions in Greenland ice sheet.
<table>
<thead>
<tr>
<th>Region No.</th>
<th>Density [kg/m³]</th>
<th>Correlation</th>
<th>Density [kg/m³]</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>383.7±50.9</td>
<td>0.5~0.8</td>
<td>522.65±23.6</td>
<td>0.7~1.0</td>
</tr>
<tr>
<td>2</td>
<td>483.1±23.4</td>
<td>0.8~1.0</td>
<td>389.74±38.9</td>
<td>0.6~1.0</td>
</tr>
<tr>
<td>3</td>
<td>485.5±48.2</td>
<td>0.6~0.8</td>
<td>538.61±88.2</td>
<td>0.3~0.7</td>
</tr>
<tr>
<td>4</td>
<td>567.9±42.0</td>
<td>0.8~0.9</td>
<td>764.57±43.4</td>
<td>0.8~0.9</td>
</tr>
<tr>
<td>5</td>
<td>596.2±34.1</td>
<td>0.8~1.0</td>
<td>688.15±28.5</td>
<td>0.8~1.0</td>
</tr>
<tr>
<td>6</td>
<td>506.0±16.4</td>
<td>0.8~1.0</td>
<td>299.15±11.2</td>
<td>0.8~1.0</td>
</tr>
<tr>
<td>7</td>
<td>-134±109.4</td>
<td>-0.3~0</td>
<td>211.46±47.6</td>
<td>0.3~0.7</td>
</tr>
<tr>
<td>8</td>
<td>489.5±101.1</td>
<td>0.3~0.9</td>
<td>363.60±25.2</td>
<td>0.7~1.0</td>
</tr>
<tr>
<td>9</td>
<td>-220.9±25.2</td>
<td>-0.6~0</td>
<td>97.65±38.8</td>
<td>-0.4~0.4</td>
</tr>
</tbody>
</table>

Table 4.2 Estimated nominal density based on inter-annual time series from GRACE and Envisat data (January 2003 to December 2009) in 9 selected regions in Greenland ice sheet. Column 2 and 3 are related to the elevation change with topographic effect removed by the use of DEM. Column 4 and 5 are associated with elevation change with topographic effect corrected by modified collinear method.

4.6.3 High Resolution Inter-annual Mass Variations over Southwest GrIS

GRACE-derived mass variation is at much coarser resolution (>330 km) than radar or laser altimetry-measured elevation change. However, altimetry-measured elevation change cannot be converted into mass variation without the exact knowledge of an appropriate density. Previously, we apply the same Gaussian smoothing to Envisat data in order to be commensurate with spatial resolution of the GRACE data. However, smoothing degrades the spatial resolution of altimetry data. Here, with the estimated density, we can convert altimetry data without Gaussian smoothing to obtain high spatial resolution inter-annual mass variations in the GrIS. Figure 4.34(a) shows, in February 2009 over region 6, the inter-annual component of mass in EWH from GRACE data with a Gaussian smoothing of 300 km. At the same time and same region, with the estimated density value, Figure 4.34(b) shows the inter-annual components of mass in EWH from Envisat data associating with modified collinear method; similar pattern can be observed, i.e., mass loss in the bottom of South GrIS and slight mass gain in the middle along the central ridge. Significant differences can also be seen from Figure 4.34, i.e., slightly positive mass change around region with latitude of 70°N and longitude of 316°E from GRACE but negative mass change from Envisat. It is apparent that inter-annual anomalies with higher spatial resolution are obtained from Envisat data. It should be noted that the quality of signal at the edge in Figure 4.34(b) might be relatively poor because of the possible effect from steep terrain or ice dynamics. With the whole time span, this would provide insights into more details on the spatial and temporal inter-annual variability of ice sheet mass change.
Figure 4.34 Comparison between the inter-annual mass variations from GRACE (a) and Envisat data using modified collinear method (b) in EWH in Feb 2009 over region 6.
Chapter 5: Antarctic Ice Sheet Mass and Elevation Changes

5.1 Introduction

The AIS is the largest store of fresh water on the Earth, with an area of ~14 million km$^2$ and a volume of ~27 million km$^3$ including ice shelves (Lythe et al., 2001; Fretwell et al., 2013). It receives widespread attention for global climate change, present-day sea level rise, as well as the advances in climate modeling and space-based technologies (Helsen et al., 2008; Hana et al., 2013; McMillan et al., 2014). During the last decade, the AIS appeared to have experienced accelerating mass loss with continuous ice dynamic thinning concentrating on its West coastal regions (Rignot et al., 2008; McMillan et al., 2014). It is unclear whether decadal or longer variations could influence the reported estimated rate of AIS mass balance acceleration during the last decade.

Since the last decade, great advances have been achieved on modeling the surface mass balance (SMB) of the AIS, although further improvement may be needed (van Wessem et al., 2014). Combining SMB and the portion of ice flux, the so-called mass budget method has become one of the three different approaches commonly used to investigate the mass variations of the AIS.

As the second method, satellite gravimetry, GRACE temporal gravity field solutions can be used to directly derive mass change since April 2002 (Tapley et al., 2004). However, estimates of rates of mass variations over the AIS at varying time periods differ widely in earlier studies, as summarized by van den Broeke et al. (2011), suffering from the large uncertainties of GIA, and being partly contaminated by the inter-annual mass variations because the study periods are relatively short. GRACE-derived mass variations are also limited by its coarse spatial resolution of ~300 km.

The third method, satellite altimetry can measure the elevation change of ice sheets. With corresponding proper density, the mass variations could be inferred. Previous studies obtained elevation change retrieved by different retrackers, i.e., ice-1 or ice-2 retracker over the AIS (Lee et al., 2012; Horwarth et al., 2012; Su et al., 2015), no comparison of elevation changes retrieved by both retrackers has been conducted, and no comparison of elevation changes with or without external digital elevation model has been investigated so far. Moreover, different assumptions of densities are adopted by earlier studies to convert altimetry-observed elevation change into mass change, as provided in Table 5.1.
A single density value of 350 kg m\(^{-3}\) is generally adopted by most researchers for firm layers over East AIS and parts of West AIS where elevation change is dominated by snowfall accumulation. Pure ice density is applied for coastal West AIS, where ice flow is believed to be the dominant process. In contrast, McMillan et al. (2014) applied pure ice density to estimate mass variation of Totten Glacier in East AIS. More complicated assumptions are applied by Riva et al. (2009), i.e., the outputs of a density model ranging from 320 kg m\(^{-3}\) to 450 kg m\(^{-3}\) for most regions in the AIS, ice density for regions with ice flux, and 600 kg m\(^{-3}\) for the region of Kamb ice stream. As a consequence, different processing about altimetric data and different density assumptions may cause discrepancies between the inferred mass changes, even for the same data within the same time period.

<table>
<thead>
<tr>
<th>Mission</th>
<th>Time period</th>
<th>Density assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Davis et al. (2005)</td>
<td>ERS-1/2</td>
<td>1992/05 – 2003/05</td>
</tr>
<tr>
<td>Zwally et al. (2005)</td>
<td>ERS-1/2</td>
<td>1992/04 - 2001/04</td>
</tr>
<tr>
<td>Wingham et al. (2006a)</td>
<td>ERS-1/2</td>
<td>1992/10 - 2003/02</td>
</tr>
<tr>
<td>Helsen et al. (2008)</td>
<td>ERS-2</td>
<td>1995 - 2003</td>
</tr>
<tr>
<td>Pritchard et al. (2009)</td>
<td>ICESat</td>
<td>2003 - 2007</td>
</tr>
<tr>
<td>Riva et al. (2009)</td>
<td>ICESat; GRACE</td>
<td>2003/03 - 2008/03</td>
</tr>
<tr>
<td>McMillan et al. (2014)</td>
<td>CryoSat-2</td>
<td>2010-2013</td>
</tr>
</tbody>
</table>

Table 5.1 Density assumptions adopted in earlier studies.

- \(a\) For East AIS.
- \(b\) For the firm layers over the AIS.
- \(c\) For basins dominated by ice dynamics in West AIS.
- \(d\) For East AIS and part of West AIS without ice dynamics.
- \(e\) For basins in the coastal region dominated by ice dynamics.
- \(f\) For Kamb ice stream.
- \(g\) For regions dominated by ice dynamics in West AIS including Kamb ice stream and Totten Glacier.

In this chapter, the surface mass variations are investigated using the Regional Atmospheric and Climate Model RACMO2/ANT in two different versions, 2.1 and 2.3. The total mass variations are estimated using GRACE RL05 monthly gravity field solutions from CSR during years 2003-2014. The elevation change from Envisat during years 2003-2009 is analyzed, especially the comparison of elevation changes from both retrackers designed for continental ice sheets, and the comparison of elevation change with and without external DEM. Furthermore, we revisit the inter-annual anomalies of mass change from GRACE and elevation change from Envisat, with the objective of estimating nominal density over the AIS at the spatial resolution of GRACE data.
5.2 Surface Mass Balance of the AIS

The SMB model involves several surface physical processes over the ice sheet. Combining with the solid ice discharge across the grounding line, it is usually used to determine the total MB of the ice sheet (the so-called mass budget method). SMB at a specific location on the ice sheet can be defined as the sum of all processes that gain or lose mass (Lenaerts et al., 2012):

\[ SMB = \int_{year} (P - SU_s - RU - ER_{ds} - SU_{ds}) \, dt \]  \hspace{1cm} (5.1)

where \( P \) represents total precipitation including snowfall and rain, \( SU_s \) is surface sublimation, \( RU \) is meltwater runoff, \( ER_{ds} \) is drifting snow erosion associated with wind velocity, and \( SU_{ds} \) is the sublimation of drifting snow.

Here, we analyze and compare the SMB variations in three portions, trend, seasonal, and inter-annual components, using the outputs in two different versions, 2.1 and 2.3, from the RACMO2/ANT model (van Wessem et al., 2014) during the period from January 2003 to December 2009. Figure 5.1 depicts the annual means (2003-2009) of SMB from the two models in the AIS. It can be seen that higher SMB occurs along the coastal regions, especially over the West coastal regions; lower SMB mostly in the interior. This is consistent with figure 1 of van den Broeke et al. (2011). Besides, there are some minor differences between annual means of SMB in these two versions, i.e., the area of regions with low SMB in the East AIS.

![Figure 5.1 Annual means of SMB (2003-2009) from RACMO2/ANT in version 2.1 (left) and 2.3 (right).](image-url)
Figure 5.2 provides the trend of SMB from these two models during years 2003-2009, after applying Gaussian smoothing with a radius of 300 km and conducting the leakage reduction with the mask shown in Figure 5.3(a). Similar variations can be observable, with mass losing mainly concentrated on Antarctic Peninsula and Wilkes Land, and mass gaining mostly over AS sector. The amplitude of the trend is relatively larger over coastal regions, i.e., larger than 4 cm/yr, while for the interior, it is less than 2 cm/yr. Besides, it is noted that the trend estimates show difference mainly over coastal regions close to Weddell Sea and Dronning Maud Land. This will be further discussed in Section 5.4.
Figure 5.4 shows the corresponding uncertainties of the estimated trends of surface mass variations in Figure 5.2. It is obtained during the fitting using Eq. (4.3). We can see that over most of the AIS, especially in the interior, the uncertainties for the trends are small for both models. Compared with the uncertainty of the trend from SMB in version 2.1 (Figure 5.2(a)), the uncertainty of the trend from SMB in version 2.3 (Figure 5.2(b)) are relative larger, i.e., 2 cm/yr, over Antarctic Peninsula.

![Figure 5.4 Uncertainties of the trends of SMB from RACMO2/ANT in version 2.1 (left) and 2.3 (right) during years 2003-2009.](image)

Figure 5.5 shows the seasonal components of SMB from model 2.1 in years 2004-2008. Here Spring includes 3 months from September to November; Summer, from December to February; Autumn, from March to May; Winter, from June to August. We can see that seasonal variations with amplitude of less than 3 cm can be seen over most of AIS, except for some parts of coastal regions during certain time. From year to year, we can observe different seasonal characteristics, i.e., more negative mass can be seen in AS sector in the summer in 2004 and in the Autumn 2008; more mass gain can be seen in AS sector in the Autumn in 2006 and in the Winter 2008. Generally speaking, mass decreases in Summer and Autumn mainly over West AIS, and mass gain mostly occur in Autumn and Spring over West AIS.

Figure 5.6 shows the seasonal components of SMB from the updated model in the same years. Similarly seasonal pattern can be observed, with a slightly larger amplitude along the coastal regions in the AIS. Here we only show the patterns of seasonal variations from year to year using SMB models. No numerical result is conducted over selected regions.
Figure 5.5 Seasonal variations of SMB from RACMO2/ANT in version 2.1.
Figure 5.6 Seasonal variations of SMB from RACMO2/ANT in version 2.3.

Figure 5.7 shows inter-annual anomalies of SMB from model 2.1 in years 2004-2008. It is clear that inter-annual anomalies of SMB increase progressively from -3 cm in 2004 to
11 cm in 2006, and decrease gradually from 2006 to 1 cm in 2008 over AS sector. Figure 5.8 provides inter-annual anomalies of SMB from model 2.3 in the same time period. We can see similar pattern of inter-annual anomalies to that from model 2.1 over coastal regions where the amplitude is relatively large. In the interior of the East AIS, there are some differences, which may be associated with the update of SMB model from 2.1 to 2.3.

Figure 5.7 Inter-annual anomalies of SMB from RACMO2/ANT in version 2.1 in 2004-2008.

Figure 5.8 Inter-annual anomalies of SMB from RACMO2/ANT in version 2.3 in 2004-2008.

5.3 GRACE-derived Mass Variation

Satellite gravimetry, GRACE, measures the redistribution of mass within the Earth system since April 2002, excluding atmosphere loading, which is forward modeled using an operational numerical weather forecasting (NWF) model, the European Centre For Medium-Range Weather Forecasts (ECMWF). It cannot distinguish the mass input by snowfall and mass output by meltwater and ice flux, or other processes, such as geodynamics. Therefore, GRACE primarily observes the Earth’s total storage change.

Here, using CSR RL05 monthly gravity field solutions during the time period from January 2003 to December 2009, we investigate the total MB variations over the ice sheets in three components, the trend, the seasonal and inter-annual terms, without and with spherical harmonic coefficient of the degree 2 and order 0 ($C_{20}$) replaced by those
SLR-derived $C_{20}$ harmonics. Figure 5.9 shows the trends of mass variations derived from GRACE data during years 2003-2009, with crustal uplift corrected from GIA model (Whitehouse et al., 2012). Similar patterns, i.e., significant mass loss in the West AIS and Antarctic Peninsula and mass gain at the Kamb Ice Stream, can be seen over the AIS. The replacement of $C_{20}$ has slight impact on the trend of mass changes.

Figure 5.10 depicts the corresponding uncertainties of the trends derived in Figure 5.9. It can be found that the uncertainties are relatively smaller in the interior of the AIS after the replacement of $C_{20}$ using the estimates from SLR.

Figure 5.9 The trends of mass variations from GRACE data during 2003-2009 without the replacement of $C_{20}$ from CSR (a) and with the replacement of $C_{20}$ using estimates derived from SLR (b). The GIA correction is applied from W12a model.
Figure 5.10 The uncertainties of trends of mass variations from GRACE keeping $C_{20}$ (a) or with them replaced by those derived from SLR (b).

Figure 5.11 and 5.12 depict the seasonal mass variations derived from GRACE data without and with $C_{20}$ replaced by those derived from SLR. We can find that amplitude of seasonal mass variations in the interior of the AIS is small, i.e., less than 3 cm over most regions. This is consistent with snowfall accumulation in small quantity in the interior. Significant differences between seasonal mass variations without and with $C_{20}$ replaced by those derived from SLR can be seen during most of periods.

Figure 5.13 and 5.14 show inter-annual anomalies of mass changes derived from GRACE data without and with $C_{20}$ replaced by those derived from SLR. During 2004-2008, very similar pattern of inter-annual anomalies can be seen over most regions in the AIS from both figures. For instance, the inter-annual anomalies increase from -2 cm in 2004 to 8 cm in 2006 over AS sector. The inter-annual anomalies are relatively small in the interior, compared with those in coastal regions in the AIS, especially in year 2006. Small differences (less than 2 cm) between inter-annual anomalies derived from GRACE data without and with $C_{20}$ replaced by those derived from SLR can be seen over some regions, i.e., Wilkes Land.
Figure 5.11 Seasonal mass variations from GRACE data without replacing \( C_{20} \) derived from SLR.
Figure 5.12 Seasonal mass variations from GRACE data with replacing $C_{20}$ derived from SLR.
5.4 Envisat-observed Elevation Change

Satellite radar altimetry is one of the most powerful tools for ice sheet observations. Previously, Wingham et al. (1998) estimated the average elevation in the interior of the AIS fell by 0.9±0.5 cm/yr from 1992 to 1996, using 5 years ERS measurements covering 63% of the grounded AIS. Davis et al. (2004) stated a five-year trend of 0.4±0.4 cm/yr in the whole AIS, with trends of 1±0.6 cm/yr in the East AIS and -3.6±1.0 cm/yr in the West AIS, using ERS-2 satellite radar altimetry during the period from June 1995 to April 2000. Lee et al. (2012) estimated that the net mass change rates are -47±8 Gt/yr during 2002-2006, -80±4 Gt/yr during 2007-2009 over Amundsen Sea (AS) sector, using elevation change retrieved by ice-1 retracker from Envisat data together with the estimated firm density and outputs from SMB models. Flament and Remy (2012) investigated the dynamic thinning of Antarctic glaciers, using the elevation change retrieved by ice-2 retracker from Envisat data during the period from September 2002 to November 2010.

As mentioned in Section 3.3, in order to generate the time series of elevation change from radar altimetry without passes exactly repeated, we need to remove topographic effect.
Here we first compare the time series of elevation changes with surface gradient correction by the use of DEM and by modified collinear method, using Envisat data retrieved by ice-1 retracker. We then compare the elevation changes retrieved by ice-1 and ice-2 retrackers, with topographic effect removed by modified collinear method.

5.4.1 Along Track Repeat Analysis with and without Use of DEM over the AIS

Two methods are used to remove the topographic effect separately. As the first approach, we use two DEMs. The DEM (grid spacing of 1 km) which combines ERS geodetic phase data and ICESat data (Bamber and Gomez-Dans, 2005) is used to correct the surface gradient. Also, the DEM generated through the ICESat data during the time period from February 2003 to June 2005 for the AIS (DiMarzio et al., 2007) is used to correct the surface gradient. It has a grid interval of 500 m, covering all of Antarctica north of 86°S. The elevation provided by the DEM is relative to the WGS 84 Ellipsoid. As the second approach, the modified collinear method removes the topographic effect by fitting a bi-quadratic surface directly using Envisat data during the time period from January 2003 to June 2010. Here, we compare RMS of elevation change time series after the surface gradient in three cases, as provided by Figure 5.15. It can be observed that RMS of elevation change time series after removing the topographic effect using modified collinear method are less than 0.5 m over most regions in the AIS; over parts of Antarctic Peninsula and coastal regions in the AIS, RMS are much larger. It also can be seen that RMS of elevation changes after correcting surface gradient by using DEM with grid interval of 1 km are less than 1 m, primarily over the interior of the AIS; over coastal regions and parts of AP, RMS are much larger than those shown in Figure 5.15(c). RMS of elevation changes after correcting surface gradient by using DEM with grid spacing of 500 m are the largest of all shown in three cases.

![Figure 5.15 RMS of elevation changes after surface gradient correction by use of DEM with grid spacing of 1 km (a), with grid interval of 500 m (b), and by modified collinear method (c).](image-url)
The trends of elevation changes corresponding to the three cases are also computed, as shown by Figure 5.16. The trend of elevation change in Figure 5.16(a) is observable primarily in the interior of the AIS; negative trend in the AS sector is recognizable; it is noisy over coastal regions. In Figure 5.16(b), the trend of elevation change is too noisy. The trend of elevation change obtained from the modified collinear method is much smoother; negative trends can be well observed in the AS sector and Totten Glacier; positive trends can be seen in the Kamb Ice Stream and some parts of coastal regions in East AIS; weakly positive trend can be seen in the interior.

![Figure 5.16](image)

Figure 5.16 Trends of elevation changes obtained separately after using DEM with grid spacing of 1 km (a), with grid spacing of 500 m (b), and the modified collinear method (c) to correct surface gradient.

Figure 5.17 illuminates the uncertainties (obtained during the fitting by using Eq. (4.3)) corresponding to the trends shown in Figure 5.16. It is clear that the uncertainty for the trend in Figure 5.16(c) is the smallest of those in all three cases, typically less than 3 cm/yr over most regions in the AIS. The uncertainty of the trend in Figure 5.16(a) is relatively small in the interior of the AIS; it is relative large in coastal regions. In Figure 5.16(b), the uncertainty of the trend is much larger, especially over the West Antarctica and its coastal regions.
Figure 5.17 Uncertainties (computed during the fitting) of trends of elevation changes obtained separately after using DEM with grid spacing of 1 \ km \ (a), 500 \ m \ (b), and the modified collinear method (c) to correct surface gradient.

5.4.2 Comparison of Elevation Change Retrieved by Ice-1 and Ice-2 Retrackers

We applied the along-track repeat analysis over the AIS, with surface gradient corrected by fitting a quadratic surface at every 350 \ m \times \ 2 \ km \ box \ (along-track \ sampling \ distance \times \ cross-track \ repeat \ band) \ directly \ using \ Envisat \ data \ during \ the \ period \ from January 2003 to June 2010. In order to assess the possible differences between the two retracking algorithms, we mainly investigate the trends of elevation changes in three cases; (1): elevation change from the ice-1 retracking algorithm with only backscatter analysis; (2): elevation change from the ice-2 retracking algorithm with solely backscatter analysis; (3): elevation change from the ice-2 retracking algorithm with corrections from changes in backscatter and waveform shapes parameters.

As the valid observations from both retracking algorithms, the ice-1 and the ice-2, are not completely the same, the number of effective observations in the box for both algorithms is depicted in Figure 5.18. We can see that for both algorithms, over most of the AIS, the number of valid measurements in the box is larger than 65, while the valid observations are relative sparse in coast region with mountains such as Princess Martha, Astrid Coast, Lars Christensen Coast, and Antarctic Peninsula. Regions with sparse observations are more obvious for the ice-2 algorithm. Based on these efficient observations within each box, by performing least squares adjustment, a bi-quadratic surface can be fitted. The time series of elevation change are then formed after removing the topographic effect and further applying corrections from changes in backscatter or backscatter together with waveform shape parameters.
As the ground tracks are not exactly repeated due to orbit drift, the individual measurements are observed at different spots. In order to form time series of elevation change, the topographic effect needs to be taken into account. To illustrate the effectiveness of surface gradient correction, we compare RMS of elevation change before and after the correction separately for the ice-1 and ice-2 retracking algorithms, as provided in Figure 5.19. For both algorithms, before surface gradient correction, the RMS of elevation change are relatively large, with magnitude of about 1m for the interior and a range from 3 m to 7 m for coast region. After the correction, the RMS become much smaller (<0.5 m and <0.7 m in the interior for the ice-1 and ice-2 algorithms, respectively), especially in coastal region, although it is slightly larger in the coast region relative to that in the interior of the AIS. On one hand, it is related to higher temperature and accumulation along the margin of the AIS (Ligtenberg et al., 2012). On the other hand, the altimeter might have poor performance over regions with rugged terrain. It also can be seen that the RMS of elevation change (after the surface gradient correction, Figure 5.19(b)) from the ice-1 algorithm are slightly smaller than these (Figure 5.19(d)) from the ice-2 algorithm in the interior. In addition, different from surface gradient correction applied by Lee et al. (2012) in the AS sector and Su et al. (2015) in the Greenland, here no external DEM is used, and the valid observations are not averaged to regular grids, which may degrade the spatial resolution.
As mentioned by Davis and Ferguson (2004) that the correlation between elevation changes and changes on backscattered power will have an impact on the seasonal, inter-annual, and long term elevation change. Thus, a linear relation between elevation changes and changes on backscattered power is usually assumed. By linear regression, the height change components, which are linearly related to changes on backscattered power, can be reduced. Using the data retrieved by the ice-2 algorithm, Horwath et al. (2012) and Remy et al. (2012) assumed a linear relation between elevation change and three waveform parameters including backscattered power, the leading edge width and the second part of trailing edge slope, to reduce the height artifact. Here we depict the correlation between elevation changes and changes on backscattered power, based on the data retrieved by the ice-1 algorithm, and the correlations between elevation changes and changes on three waveform parameters provided by the ice-2 algorithm, respectively. From Figure 5.20(a) and Figure 5.20(b), we find that for both algorithms, positive correlation can be seen over most of the AIS. The percentage of positive correlation on the order of 0.6 is about 42.6% for the ice-1 algorithm and 30.8% for the ice-2 algorithm, mainly in the interior of East AIS. Weakly positive and negative correlations can be found along the coastal region and
Antarctic Peninsula for both algorithms, which is similar to the pattern shown by Davis and Ferguson (2004) in their Figure 2(a) using data from ERS-2. Compared with data from ERS-1/2, data from Envisat shows a better spatial coverage, but the correlation values between elevation change and changes in backscatter coefficients are smaller over some regions. Davis and Ferguson (2004) also reported that the correlation values they calculated from ERS-2 data were smaller than that from ERS-1 over some regions. This may be associated with the different observational time periods, different instruments, and different retracking algorithms. Figure 5.20(c) maps the correlation between elevation changes and changes on the leading edge from the ice-2 algorithm. Strongly negative correlations can be found over the covered AIS, with 86% of correlations less than −0.6. Similar map was shown by Remy et al (2012) in their Figure 3(a). Weakly negative correlations can be mainly found along the coastal region. Figure 5.20(d) illustrates the correlations between elevation changes and changes on the second part of trailing edge slope. There are some weak negative correlations but no strong correlations can be observed.

In Figure 5.21(a) and Figure 5.21(b), for both algorithms, we represent the correlation gradient, which reflects a linear relation with elevation changes versus changes on backscattered power. It is noted that case 1 and case 2 involve only backscatter analysis. It is apparent that the high correlation does not necessarily correspond to a high correlation gradient. Comparing with that for the ice-1 algorithm, the correlation gradient for the ice-2 algorithm is more positive over some regions in the AIS. Over all the regions, the mean of the correlation gradient is 15.5 cm/dB for the ice-1 algorithm, and 24.11 cm/dB for the ice-2 algorithm. In addition, for both algorithms, correlation gradients show some patterns different from that provided by Davis and Ferguson (2004) using the data from ERS-2, especially over basin B-C where the largest correlation gradient on the order of 50 cm/dB occurred in their Figure 2(b). These differences could be due to differences between instruments for ERS-2 and Envisat, differences in the retracking algorithms, and also different period of observations. For case 3, the correlation gradient is provided in Figure 5.21(c). After adding waveform shape parameters, we find that the correlation gradient (for backscatter coefficients) is reduced over regions where highly positive correlation gradient occurs in Case 2, which makes the mean of the correlation gradient reduced to 11.6 cm/dB, slightly smaller than that of the correlation gradient in Case 1.
Figure 5.20 Correlation between elevation change and backscatter change from (a) the ice-1 algorithm (b) from the ice-2 algorithm (c) Correlation between elevation change and change on the leading edge width from the ice-2 algorithm (d) Correlation between elevation change and the trailing edge slope change from the ice-2 algorithm.

Figure 5.21 Correlation gradient representing the magnitude of change in elevation versus backscatter change (a) for the ice-1 algorithm (b) for the ice-2 algorithm.
Furthermore, differences between the trends of elevation changes before and after applying the correction from backscattered power or from backscattered power together with waveform shape parameters are shown in Figure 5.22. We can see that the majority of differences between the trends occur in the interior of East AIS. Figure 5.22(a) show a slightly small differences with a range of $\pm 6$ cm/yr from data retrieved by the ice-1 algorithm. Figure 5.22(c) show the most obvious differences with a range of $\pm 10$ cm/yr. Together with Figure 5.21, it can be clearly seen that large differences between the trends generally occur in areas with large correlation gradient values, i.e., the region close to Dome C, while small trend differences occur in areas with small correlation gradient values. In addition, the trends of the time series of backscatter changes for both algorithms are shown in Figure 5.23. Very similar spatial patterns can be observed for the trends of backscatter change time series from both algorithms; positive trends mainly occur around the longitude of 30°E and the latitude from 81.5°S to 75°S, the longitude of 90°E and the latitude from 80°S to 67°S, and the longitude of 150°E and the latitude from 80°S to 69°S; negative trends appear around the longitude of 55°E and the latitude about 75°S, the longitude of 120°E and the latitude about 77°S and 70°S. For both algorithms, almost 95% of the trends on backscatter changes falls within a range of $\pm 0.3$ dB/yr. It can be noted that the region where large trends on backscatter changes occur experiences the large trend change of elevation changes after the correction from backscattered power or together with waveform shape parameters. As stated by Davis and Ferguson (2004) that the combination of the correlation between changes in elevation backscattered power, the corresponding correlation gradient and the trend of backscatter changes will play a vital impact on changes of the trend of elevation changes after the correction from backscatter changes. On the other hand, the spatial pattern of the trend of changes in backscattered power from Envisat in Figure 5.23 is different from that in Figure (5) of Davis and Ferguson (2004) using ERS-2 data. We think this may be due to different observation time period, different instruments, and different retracking algorithms.

Figure 5.22 Spatial plot of differences between the trends of elevation changes before and after backscatter correction (a) for case 1 (b) for case 2 (c) case 3.
Figure 5.23 Trend in backscatter coefficient change (a) from the ice-1 algorithm, (b) from the ice-2 algorithm.

After removing the height change component, which is linearly related to changes on backscattered power or waveform shape parameters, the trends of elevation changes for the three cases are calculated, as provided in Figure 5.24. By visual inspection, similarly spatial pattern can be observed in all the three cases. In West AIS, dramatic thinning of the Pine Island Glacier and the Thwaites Glacier can be well recognized; an increase on the small upstream part of Kamb Ice Stream can also be visible. In East AIS, a decrease on Totten Glacier can be seen; a slightly positive trend can be perceived over most of the interior of the East AIS. Although the periods of study data are different from that adopted by other research, the trend of elevation changes exhibits similar spatial pattern, as provided in their Figure 2(a) by Horwath et al. (2012) and in their Figure (1) by Flament and Remy (2012).
In order to evaluate the discrepancies on the trends of elevation changes obtained from both algorithms, and the effect, which is caused by adding the correction from waveform shape parameters for the ice-2 algorithm, on the trend of elevation changes, the following terms are calculated: (a) the trend of elevation change from Case 1 subtract that from Case 2; (b) the trend of elevation change from Case 3 subtract that from Case 2; (c) the trend of elevation change from Case 1 subtract that from Case 3. From Figure 5.25(a), only with backscatter analysis, a difference of the trend is around a range of ±2 cm/yr over most of East AIS. Along the latitude of 70°S and the longitude from 90°E to 130°E, a difference of −5 cm/yr can be found. Similar disagreements can be seen in Figure 5.25(b), which is consistent with the result shown by Remy et al. (2012) in their Figure 7(b). It is noted that here we do not perform interpolation so that the result is noisy over the coastal regions. Nevertheless, it can be clearly found that the differences between Case 1 and Case 3 in absolute value is less than 1 cm/yr over most of East AIS, especially in the interior. Over coastal regions, the difference is slightly larger due to the effect from rugged ice topography. That is, the trend of elevation changes with only backscatter analysis from the ice-1 algorithm agree with that of elevation changes with corrections from backscatter coefficients and waveform shape parameters from the ice-2 algorithm.
5.5 Inter-annual Anomalies Combining GRACE and Envisat in the AIS

Assuming that mass and elevation changes related to the same geophysical process could be well detected by GRACE and altimetry separately over the AIS, the combination of GRACE-derived mass variations and Envisat-observed elevation changes will enable us to estimate nominal density. Lee et al. (2012) used an empirical method, comparing seasonal amplitudes of elevation changes from altimetry and mass changes in EWH from GRACE, to estimate firn densities during 2002–2006 and 2007–2009 over the AS sector. Su et al. (2015) estimated nominal densities ranging from 383.7±50.9 to 596.2±34.1 kg m$^{-3}$ over selected regions over the GrIS, combining inter-annual anomalies of mass change from GRACE and elevation change from Envisat. Horwarth et al. (2012) compared the two inter-annual anomalies from 10-d CNES/GRGS GRACE and Envisat data, and found consistent pattern mostly along the coastal regions over the AIS. Thus, at least over some regions, it is possible to estimate nominal density by combining the two data sets represented by their respective inter-annual anomalies.

This Section presents an updated joint analysis of inter-annual anomalies of mass change from GRACE (CSR RL05 data) and elevation change from Envisat during January 2003 to December 2009. The two inter-annual anomalies are compared, and their temporal correlation is computed over the AIS. At chosen regions, nominal densities are estimated using the inter-annual anomalies from GRACE and Envisat data.

In order to make sure that the two inter-annual anomalies are correct, we output the linear trends from time series of GRACE-derived mass changes and Envisat-observed elevation changes separately. To be commensurate with GRACE data, i.e., with the same spatial resolution, the linear trend from Envisat data is averaged to the same regular grids as that from GRACE data and corrected crustal uplift caused by GIA (Whitehouse et al., 2012). The same Gaussian smoothing with a radius of 300 km and leakage reduction with the ice
mask shown in Figure 5.3(b) are then applied. As provided in Figure 5.26, similar pattern can be observed on both linear trends, with significant mass loss and elevation decrease occurring over the AS sector, moderate mass loss and elevation decrease in Wilkes Land and Totten Glacier, mass gain and elevation increase at Kamb Ice Stream and some parts of coastal regions in East AIS. The main difference is over Antarctic Peninsula where significant mass loss can be seen from GRACE and no reliable data can be retrieved from Envisat because of the poor performance of radar altimetry over rugged ice topography.

![Figure 5.26 Linear trends from GRACE in EWH (a) and Envisat (b) in height with GIA corrected by W12a GIA model.](image)

5.5.1 The Two Inter-annual Anomalies and their Correlation in the AIS

The inter-annual anomalies of mass change from GRACE in ice equivalent height and elevation change from Envisat in height are compared during the 2004–2008. As shown in Figure 5.27, large inter-annual anomalies mostly occur along coastal regions. This is mainly associated with snowfall variability (Ligtenberg et al., 2012). For GRACE-derived mass change over the AS sector, the pattern of mass gain gradually from 2004 to 2006 (Figure 5.27(a) – (c)), and that of mass loss step by step from 2006 to 2008 (Figure 5.27(c) – (e)) are evident, and agree well with the pattern shown by Envisat-observed elevation change during the same time period. Over part of Wilkes Land and the Weddell Sea coast, similar pattern can also be detected by both technologies. It is apparent that Envisat-observed inter-annual anomalies range from 5 cm to more than 10 cm along the coastal regions, with amplitudes slightly larger than those of GRACE-derived inter-annual anomalies, partly due to different densities. In contrast to the coastal regions, smaller inter-annual anomalies with amplitudes mostly less than 4 cm can be found in the interior.
In order to assess the similarity between the two inter-annual anomalies, temporal correlation between them is computed over each regular grid covered by both datasets, as depicted by Figure 5.28. Highly positive correlations with coefficient larger than 0.6 are mainly found over the West AIS, regions around Princess Martha Coast, and part of Dronning Maud Land and Wilkes Land, occupying more than 40% of the covered area. Weakly positive correlations mostly appear in the East AIS and a small part of West AIS. Negative correlations occur in the East AIS, with a percentage of 21%. For parts of Antarctic Peninsula, no correlations are computed because of the limitation from rugged ice topography from Envisat altimetry. The correlation map is similar to that in Figure 5 of Horwath et al. (2012) who adopted Envisat data retrieved by ice-2 retracker and GRACE 10-day gravity field solutions from CNES/GRGS. Significant differences are that, weakly positive correlations are found over regions with longitude from 90°E to 135°E and latitude from 67°S to 73°S; negative correlations are mostly concentrated on regions near to Dome C Lake District, Lake Vostok, and region with longitude from 50°E to 60°E and latitude from 70°S to 75°S. Comparing the correlation map with published locations of subglacial lakes (black triangles indicate subglacial lakes discovered by radio-echo sounding and black dots depict subglacial lakes invented by satellite altimetry), we find that negative correlations occur mainly in regions where subglacial lakes cluster, although there is no direct evidence that subglacial lakes may interrupt the inter-annual variations of either mass or surface elevation.
5.5.2 Estimation of Nominal Density

Following the correlation map, we select 14 regions with strong positive, moderate positive as well as negative correlation, as indicated by the blue polygons numbered from 1 to 14 in Figure 5.28. Over these regions, we compare the inter-annual anomalies of mass changes from GRACE in EWH and elevation changes from Envisat in height, as shown in Figure 5.29. It is apparent that the amplitudes of inter-annual signals detected by GRACE (blue dotted lines) are less than 10 cm in EWH for all the regions, with smaller amplitudes shown in the interior, i.e., region 3, 4, 5, and 8. For altimetry, inter-annual signals with amplitude about or less than 10 cm can be found over region 1 to 11, with smaller amplitude of less than 5 cm shown in regions 3, 4, 5, and 8 in the interior of East AIS, whereas amplitudes larger than 10 cm can be seen over region 12, 13, and 14 in the West AIS. This is consistent with the mean SMB from RACMO2/ANT over AIS showing that coastal West AIS experiences higher accumulation rate (van den Broeke et al., 2011). Besides, the two inter-annual time series show similar pattern over most of regions except for regions 3, 5, 8 where negative correlations occur. Using monthly outputs from RACMO2/ANT, the inter-annual anomalies of surface mass variations from SMB models in two different versions, v2.1 (cyan dotted line) and v2.3 (pink dotted line), are extracted over each regions, as shown in Figure 5.29. Over most regions, the inter-
annual anomalies from SMB models are consistent with those from GRACE data. This suggests that the same geophysical process dominates the signals detected by satellite gravimetry and altimetry over most regions. Thus, the combination of the two inter-annual anomalies provides us an opportunity to estimate the nominal density of the portion related to inter-annual mass and elevation changes by purely using geodetic observations.

Based on the two inter-annual time series of mass change and elevation change, and the assumption that at the inter-annual timescale the firn compaction component, which alters the surface elevation but not mass change, is small or negligible, the nominal density value can be estimated by linear regression (Eq. 4.6) in each region, as provided in Table 5.2. For 4 regions all with correlation coefficients larger than 0.7, which to some extent confirms that both technologies detect the same geophysical process, the estimated nominal densities range from $317.49 \pm 24.2$ to $479.81 \pm 25.3 \text{ kg m}^{-3}$.

Previously, Horwath et al. (2012) identified an event of excess snowfall with mass of $82 \pm 31 \text{ Gt}$ in the West AIS in September/October 2005. Lee et al. (2012) reported the density change over AS sector empirically using the ratio of amplitudes of seasonal variations. Here we compute the surface densities based on the two inter-annual anomalies in region 12 (AS sector) over two different time periods separately. The estimated nominal density is $450.17 \pm 28.8 \text{ kg m}^{-3}$ between 2003 and 2006, and $812.45 \pm 44.3 \text{ kg m}^{-3}$ between 2007 and 2009, which agrees well with those provided by Lee et al. (2012) in their Table 2 using Release 4 GRACE data from CSR and old Envisat data, with small discrepancy mainly caused by different versions of GRACE solutions and Envisat data, not exactly the same study region, and different time period of data. Furthermore, we compute temporal variations of nominal densities over 4 regions with correlation larger than 0.7, following the procedures: (1) select the two inter-annual time series covering the same period with a length of 27 months, i.e., both time series are from July 2003 to September 2005, nominal density can be estimated by linear regression; (2) with an interval of one month, adjust both time series, i.e., from August 2003 to October 2005, estimate nominal density again; (3) repeat step (2) until the end of both time series. The selection of 27 months is arbitrary but under the consideration that the nominal density can be estimated reliably and the pattern of density change can be revealed well. As shown in Figure 5.30, over regions 1, 11, and 13, the estimated nominal densities are close to the density of snow. Large uncertainties (red shadings) can be seen in the nominal density variations in region 1. Over region 12, a slight decrease of nominal density occurs at the end of 2005, and an abrupt increase of nominal density starts at 2008 can be observed. This confirms that the surface mass change is dominated by snow accumulation during 2003–2006, and by ice flow during 2007–2009, and sheds light on the surface density change versus time over the AS sector. Empirically, we know that region 12 is dominated by ice thinning. Therefore, if ice density is used to estimate mass change over this region during the whole period, it may cause inaccuracy in the total mass change. From Table 5.2, over East AIS, the estimated densities for regions with
moderate positive correlations are slightly less than snow density; for regions with negative correlations, the nominal density is not physically possible. For region 10 and 14 in West AIS, the nominal densities are slightly less than snow density.

Figure 5.29 Inter-annual time series from GRACE (blue dotted line) in EWH and Envisat (red dotted line) in height in 14 chosen regions in the AIS.
<table>
<thead>
<tr>
<th>Region No.</th>
<th>Density [kg m⁻³]</th>
<th>Uncertainty [kg m⁻³]</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>359.34</td>
<td>12.6</td>
<td>0.9-1</td>
</tr>
<tr>
<td>2</td>
<td>248.88</td>
<td>34.6</td>
<td>0.4-1</td>
</tr>
<tr>
<td>3</td>
<td>-378.6</td>
<td>58.4</td>
<td>-0.8-0</td>
</tr>
<tr>
<td>4</td>
<td>122.02</td>
<td>32.2</td>
<td>0.2-0.6</td>
</tr>
<tr>
<td>5</td>
<td>-69.95</td>
<td>23.8</td>
<td>-0.5- -0.1</td>
</tr>
<tr>
<td>6</td>
<td>159.87</td>
<td>60.3</td>
<td>0.3-0.7</td>
</tr>
<tr>
<td>7</td>
<td>126.92</td>
<td>58</td>
<td>0.3-0.5</td>
</tr>
<tr>
<td>8</td>
<td>-165.41</td>
<td>28.1</td>
<td>-0.8- -0.2</td>
</tr>
<tr>
<td>9</td>
<td>269.18</td>
<td>49.6</td>
<td>0.3-0.7</td>
</tr>
<tr>
<td>10</td>
<td>214.12</td>
<td>27.1</td>
<td>0.1-0.7</td>
</tr>
<tr>
<td>11</td>
<td>379.52</td>
<td>23.2</td>
<td>0.8-1</td>
</tr>
<tr>
<td>12</td>
<td>479.81</td>
<td>25.3</td>
<td>0.8-1</td>
</tr>
<tr>
<td>13</td>
<td>317.49</td>
<td>24.2</td>
<td>0.8-1</td>
</tr>
<tr>
<td>14</td>
<td>249.66</td>
<td>24.1</td>
<td>0.4-1</td>
</tr>
</tbody>
</table>

Table 5.2 Estimates of nominal density based on inter-annual anomalies from GRACE and Envisat data (January 2003 to December 2009) over 14 chosen regions in the AIS.

Figure 5.30 Nominal density variations over 4 regions (correlations > 0.7). The uncertainties (red shadings): absolute value of differences between nominal densities estimated from joint analysis of CSR-published gravity field solutions and Envisat data, and GFZ-published gravity field solutions and Envisat data separately.
As the inter-annual anomalies may contaminate the estimated trend because of the relatively short time period of observations, it is important to investigate the pattern of inter-annual variability in spatial and temporal scales to correctly interpret differences of trend during different time periods. The datasets from GRACE and Envisat have their own limitations, i.e., the coarse spatial resolution of \( \sim 300 \) km and the linear trend suffered from large uncertainties of GIA models for GRACE data, the limited spatial coverage, density information required, and the influence from rugged ice topography for Envisat data. With inter-annual anomalies of mass changes from GRACE and elevation changes from Envisat, in principle we are able to estimate nominal density and improve the spatial resolution of inter-annual mass variations based on density-corrected altimetry over AIS. However, weak positive and negative correlations between the two inter-annual anomalies are found over most regions in AIS, causing the estimated density physically unrealistic (< 300 kg/m\(^3\)).

On the one hand, negative correlations may be associated with the uncertainties of the datasets. Horwath et al. (2012) found negative correlation between the two inter-annual variations in most of the East AIS. They suspected possible errors in the 10-d CNES/GRGS GRACE time-variable gravity field solutions. Here we use the improved RL05 monthly GRACE gravity field solutions from CSR. In order to determine whether the signals of mass change can be detected by GRACE or not, we select three regions with negative correlations, as shown in Figure 5.31, to inspect the original time series of mass change and elevation change. As provided in Figure 5.32, the time series from GRACE data (blue dotted line) show a difference of 4–5 cm in EWH between the peak-to-local minimum values in each year for all the three regions. We think the amplitude of the signals is not too small to be detected by GRACE. From the time series of elevation change (red dotted line), seasonal variation can be observed, with the maximum of elevation in June, July, or August and the minimum in December, January, or February. The phase change from Envisat data is consistent with the seasonal elevation oscillation shown by SMB model, although elevation changes from Envisat are not perfect as we expect in 2003–2004.

On the other hand, we notice that there are phase differences between the two original time series in all the three selected regions. The mass changes revealed by GRACE have maximum value generally in March to May. The maximum elevation change from Envisat is achieved in June to August when a decrease can be seen from time series of mass changes from GRACE. If the two time series are reliable, we may suggest that different geophysical processes are detected by both technologies. As shown in Figure 5.28, the regions with negative correlation appear to be coincident with the location of clustered subglacial lakes, mostly around Lake Vostok and Dome C Lake District, although there is no direct evidence that the drainage of subglacial lakes may interrupt inter-annual anomalies of either mass or elevation change. Previously, using ERS-2 altimetric data, Wingham et al. (2006b) reported that 1.8 km\(^3\) of water was transferred to at least two subglacial lakes with a distance of 290 km during 16 months. They suggested that entire subglacial drainage basins may be flushed periodically, although subglacial
lakes are expected to have long residence times and slow circulations. Smith et al. (2009) identified 124 active subglacial lakes using elevation change from laser altimetry, suggesting that on annual and shorter timescales, water is available in many parts of AIS for exchanges between lakes or between lakes and the surrounding basal system. They also stated that over a few thousand square kilometers, distributed sources of water such as linked cavities, could supply certain amounts of water without producing surface-elevation changes detectable with ICESat data. To some extent, this may be a possible interpretation about the phase differences observed in Figure 5.32. In addition, Wright et al. (2012) modeled ice sheet flow and basal ice temperatures, by analyzing basal topography, bed roughness and radar power reflectance, they illustrated the potential of the 1000 km-long basin, the Aurora Subglacial basin, as a pathway for subglacial water drainage from the interior to coastal regions in East AIS.

Figure 5.31 Three selected regions with negative correlations (region 3, 5, and 8).
Using the outputs from the RACMO2/ANT (SMB model in the AIS; independent source) in two different versions during 2003–2009, as described in section 5.2, the inter-annual components are extracted, respectively. The correlations between the inter-annual anomalies of surface mass change from the RACMO2/ANT in v2.1 and v2.3 and of mass change derived from GRACE data are then computed separately, as shown in Figure 5.33. Over most regions in the AIS, highly positive correlation can be observed in both figures, especially over the West AIS. Over the East AIS, there are some regions with negative correlations as shown in both figures. It seems that inter-annual anomalies from the SMB model in v2.3 are positively correlated with that from GRACE data over the region close to coastline near to the Weddell Sea, where negative correlation is shown in Figure 5.33(a). Similar cases can be also observed in a small region in the South pole with longitude of −30°, and regions with longitude of 80°–90° and latitude of 67°S–70°S. The cause for the negative correlation in Figure 5.33 is unknown.
Figure 5.33 Correlation of inter-annual time series from GRACE and SMB in v2.1 (a) and in v2.3 (b).
Chapter 6: Conclusions and Future Studies

Satellite gravimetry and altimetry provide valuable measurements about mass and elevation changes of continental ice sheets, respectively. In this study, different post-processing methods are evaluated using Envisat altimetric data over continent ice sheets; joint analysis of GRACE gravimetry and Envisat altimetry is conducted, with an objective of estimating nominal density associated with measured mass and elevation changes at the inter-annual scale.

The along-track repeat analysis is utilized in processing Envisat altimetric data. For surface gradient correction, two approaches, use of DEM and collinear method, are modified to remove topographic effect due to the inexactly repeated ground tracks from Envisat over both ice sheets. Over the GrIS, by statistic analysis of the RMS of generated elevation change time series using both methods separately, the percentages of data with RMS less than 1 m are around 45% for use of DEM method and 85% for modified collinear method, respectively. Moreover, the trends of elevation change time series after applying empirical correction from backscatter analysis are computed, using both methods. Compared with the trend associated with use of DEM method, the trend of elevation change related to the modified collinear method is much smoother. And the corresponding uncertainty of the trend of elevation changes from modified collinear method is much smaller over wider regions in the GrIS. Similar comparison is also conducted over the AIS, with the result showing that the percentage of data with RMS less than 1 m is 88%, and the trend of elevation change is much smoother after removing topographic effect using modified collinear method. It is thus concluded that the modified collinear method is more effective to remove the topographic effect and generate elevation change time series from Envisat altimetry.

As to empirical correction, two retracking algorithms, the ice-1 and ice-2 retrackers, are analyzed in three cases: (1) for elevation change retrieved from the ice-1 algorithm, only backscatter analysis is used; (2) for elevation change from the ice-2 algorithm, only backscatter analysis is used; (3) for elevation change from the ice-2 algorithm, corrections from changes in backscatter coefficients and waveform shape parameters are used. With the topographic effect removed by modified collinear method, the above three cases are studied to investigate the trend of elevation change over the AIS. By differencing the trends, Case (1) and Case (2) show that a difference of the trends is around a range of ±2 cm/yr over most of East AIS; along the latitude of 70°S and the longitude from 90°E to 130°E, a difference of 5 cm/yr can be found. Similar differences
can also be found between Case (2) and Case (3). For Case (1) and Case (3), the difference of the trends in absolute value is less than 1 cm/yr over most of East AIS, especially in the interior. This confirms the conclusions in a previous study that for the ice-2 algorithm, only backscatter analysis is not enough to correct height artifacts, and waveform shape parameters are needed. It is then concluded that the trend estimates from the ice-1 algorithm with only backscatter analysis agrees well with that from the ice-2 algorithm with corrections from changes in backscatter coefficients and waveform shape parameters.

After generating time series of elevation changes from Envisat altimetry and mass changes from monthly RL05 Level 2 GRACE data published by CSR during 2003-2009, two case studies are conducted over the GrIS and AIS, with the objective of estimating nominal density of firn/ice associated with measured mass and elevation changes at the inter-annual scale, purely using geodetic observations for the first time.

In the GrIS, highly positive correlations (>=0.6) between inter-annual anomalies of mass change from GRACE and elevation change from Envisat data are found mostly over the North, West, Southwest, and parts of Southeast GrIS, accounting for 77.6% of the GrIS; weakly positive correlations occur in Southeast and Northwest GrIS; negative correlations appear in a small part of Northeast GrIS. Moreover, using the monthly outputs from RACMO2/GR (an independent source), the inter-annual surface mass variations are extracted over the GrIS. The correlation between inter-annual anomalies of surface mass change from RACMO2/GR and mass change from GRACE data are computed, showing that positive correlation occurs almost over the whole GrIS except for one spot in Southeast part; high correlation with magnitude larger than 0.7 are found over most of the GrIS; weakly positive correlation appears in Southeast and a small part of Northeast GrIS. This suggests that, at the inter-annual scale, what GRACE measures is surface mass variation in the GrIS. To some extent, it confirms the consistency between inter-annual surface mass variations and those obtained from GRACE, indicating that the inter-annual elevation changes derived from Envisat reflect the inter-annual surface mass variation in the GrIS except for the small region of negative correlation. Furthermore, assuming that the inter-annual firn compaction component, which alters the elevation change but not mass change, is negligible, nominal densities are estimated in 9 selected regions by combining the inter-annual anomalies of mass change from GRACE data and elevation change from Envisat data. The estimated density values range from 299.15±11.2 kg m⁻³ to 764.57±43.4 kg m⁻³ for 8 regions, with one value physically impossible for region 9. Based on Envisat data corrected with estimated density, the inter-annual mass variations are obtained in region 6 with a higher spatial resolution.

In the AIS, highly positive correlations with coefficient larger than 0.6 are mainly found over West AIS, regions around Princess Martha Coast, and part of Dronning Maud Land and Wilkes Land, occupying more than 40% of the covered area. Weakly positive correlations mostly appear in the East AIS and a small part of the West AIS. Negative correlations occur in the East AIS, with a percentage of 21%. Similar to the study in the
GrIS, using monthly outputs from the model RACMO2/ANT (an independent source) during the same study period, the inter-annual surface mass variations are extracted. The correlation between inter-annual anomalies of mass change from GRACE and surface mass change from the RACMO2/ANT is computed. Highly positive correlations on the order of 0.6 occur over most of the AIS; negative correlation mainly occurs around regions with longitude from 70°E to 100°E and latitude from 74°S to 82°S. Based on the two inter-annual anomalies of mass change from GRACE data and elevation change from Envisat, and the same assumption for the GrIS, nominal densities are estimated over 14 selected regions, with a range of 317.49±24.2 kg m\(^{-3}\) to 479.81±25.3 kg m\(^{-3}\) for 4 highly positive-correlated regions and values slightly less than snow density for another 4 regions of weak positive correlations as well as values physically unrealistic for all other regions of weakly positive and negative correlations.

It is concluded that the joint analysis of GRACE gravimetry and Envisat altimetry provides constraints on different density assumptions widely adopted by researchers for converting measured elevation change into mass change. This could narrow the uncertainty of mass variations of ice sheets. On the other hand, the inter-annual mass variations with high spatial resolution can be obtained by density-corrected Envisat altimetric data, which will shed lights on the inter-annual mass variations in local regions and enable the potential improvement of SMB models.

For future study, the possible assumptions for regions with negative correlations may be tested. The estimated density could be compared with those measured by in situ data, i.e., data at the summit of the GrIS. Data from the past mission, ERS-1/2, could be used to extend the time series of elevation changes, enabling the investigation of characters of elevation changes during 1995-2009. New data sets from Cryosat 2 and Altika altimetry can be analyzed over both ice sheets. Especially for Altika altimetry, the time series of elevation changes can be generated.
Bibliography


